

# The Complexity of Phrase Alignment Problems



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# The Phrase Alignment Problem

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# The Phrase Alignment Problem

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Japan to freeze aid to Russia .

# The Phrase Alignment Problem

Japan to freeze aid to Russia .

日本

冻结

向

俄

提供

援助

◦

# The Phrase Alignment Problem

Japan to freeze aid to Russia .

Pinyin

Gloss

日本 ri4 ben3 *Japan*

冻结 dong4 jie2 *freeze*

向 xiang4 *to*

俄 e2 *Russia*

提供 ti2 gong1 *supply*

援助 yuan2 zhu4 *assistance*

◦

◦

◦



# The Phrase Alignment Problem

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	<u>Pinyin</u>	<u>Gloss</u>
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冻结	dong4 jie2	<i>freeze</i>
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○	○	○



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Phrase alignments are one-to-one and onto

# The Weighted Phrase Alignment Problem

Japan to freeze aid to Russia .


	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	<i>Japan</i>
冻结	dong4 jie2	<i>freeze</i>
向	xiang4	<i>to</i>
俄	e2	<i>Russia</i>
提供	ti2 gong1	<i>supply</i>
援助	yuan2 zhu4	<i>assistance</i>
o	o	o

# The Weighted Phrase Alignment Problem

Japan to freeze aid to Russia .

0.9						

	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	<i>Japan</i>
冻结	dong4 jie2	<i>freeze</i>
向	xiang4	<i>to</i>
俄	e2	<i>Russia</i>
提供	ti2 gong1	<i>supply</i>
援助	yuan2 zhu4	<i>assistance</i>
o	o	o

# The Weighted Phrase Alignment Problem

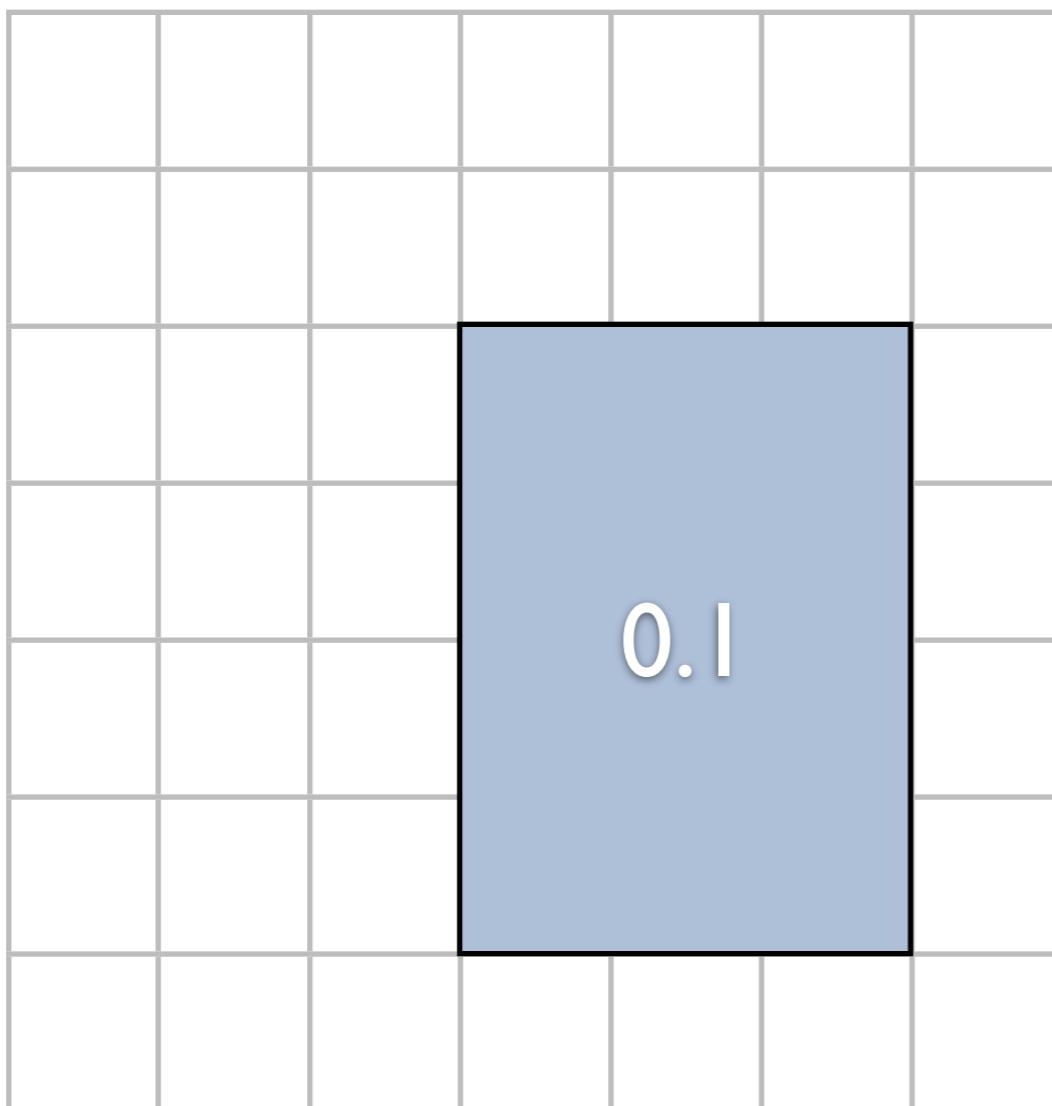
Japan to freeze aid to Russia .

	0.6					

	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	<i>Japan</i>
冻结	dong4 jie2	<i>freeze</i>
向	xiang4	<i>to</i>
俄	e2	<i>Russia</i>
提供	ti2 gong1	<i>supply</i>
援助	yuan2 zhu4	<i>assistance</i>
o	o	o

# The Weighted Phrase Alignment Problem

Japan to freeze aid to Russia .



	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	<i>Japan</i>
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提供	ti2 gong1	<i>supply</i>
援助	yuan2 zhu4	<i>assistance</i>
◦	◦	◦



# The Weighted Phrase Alignment Problem

Japan to freeze aid to Russia .

0.002

	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	<i>Japan</i>
冻结	dong4 jie2	<i>freeze</i>
向	xiang4	<i>to</i>
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Japan to freeze aid to Russia .


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日本	ri4 ben3	<i>Japan</i>
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提供	ti2 gong1	<i>supply</i>
援助	yuan2 zhu4	<i>assistance</i>
o	o	o

# The Weighted Phrase Alignment Problem

Japan to freeze aid to Russia .

0.9					
	0.6				
		0.8			
			0.9		
		0.7			
					0.9

	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	<i>Japan</i>
冻结	dong4 jie2	<i>freeze</i>
向	xiang4	<i>to</i>
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◦	◦	◦

# The Weighted Phrase Alignment Problem

Japan to freeze aid to Russia .

0.9					
	0.6				
		0.8			
			0.9		
		0.7			
					0.9

	<u>Pinyin</u>	<u>Gloss</u>
日本	ri4 ben3	Japan
冻结	dong4 jie2	freeze
向	xiang4	to
俄	e2	Russia
提供	ti2 gong1	supply
援助	yuan2 zhu4	assistance
◦	◦	◦

$$0.9 \cdot 0.6 \cdot 0.7 \cdot 0.8 \cdot 0.9 \cdot 0.9 = 0.24$$



# Applications Involving Phrase Alignments



# Applications Involving Phrase Alignments

- Inference under a phrase alignment model



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- Inference under a phrase alignment model
- Viterbi Training for a phrase alignment model



# Applications Involving Phrase Alignments

- Inference under a phrase alignment model
- Viterbi Training for a phrase alignment model
- Forced decoding for phrase-based systems



# Applications Involving Phrase Alignments

- Inference under a phrase alignment model
- Viterbi Training for a phrase alignment model
- Forced decoding for phrase-based systems
- Improved decoding for word alignment models



# Related Problems are Polynomial

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# Related Problems are Polynomial

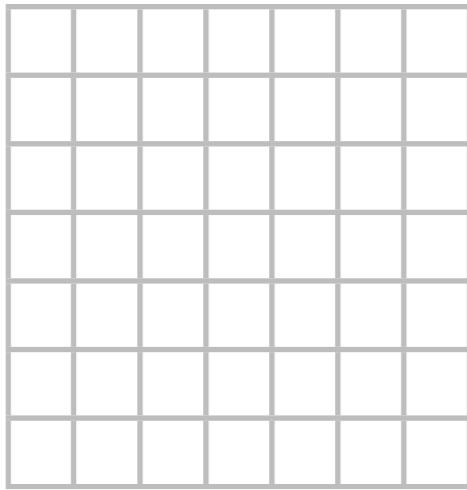
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**MATCHING:** Given segmentations, find the maximal matching.

# Related Problems are Polynomial

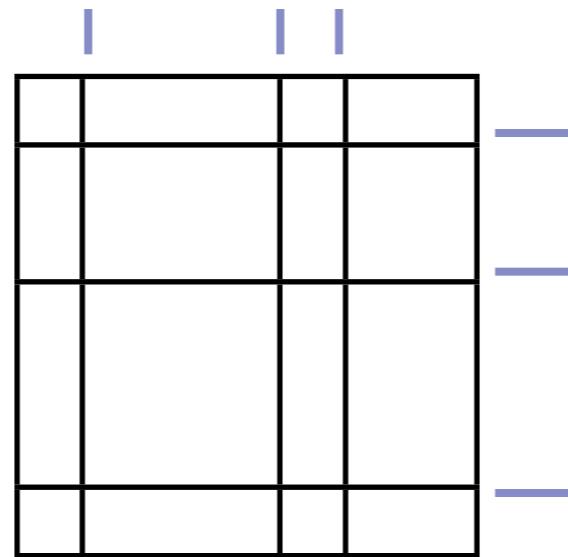
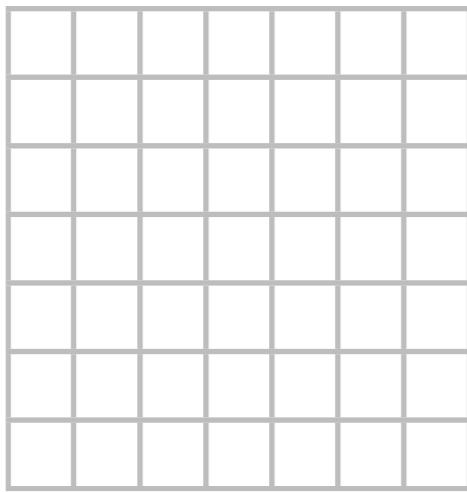
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**MATCHING:** Given segmentations, find the maximal matching.



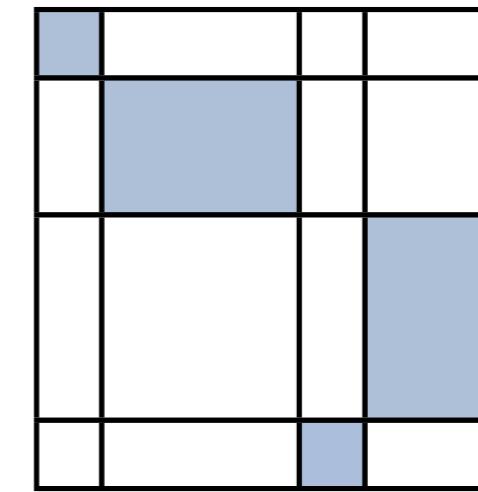
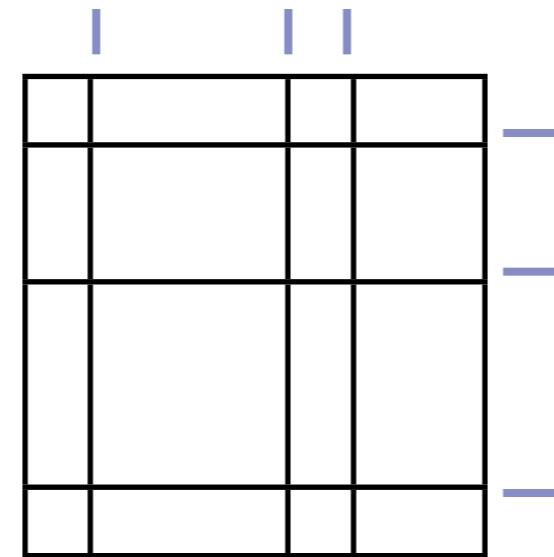
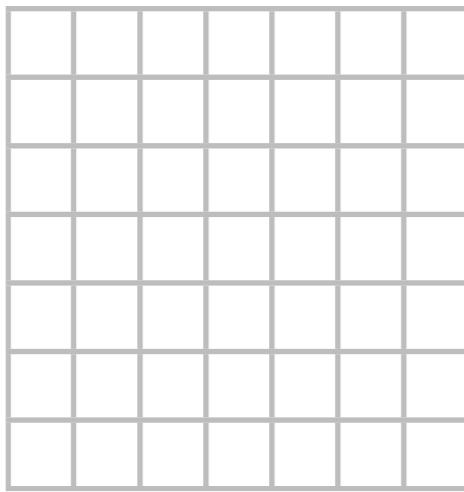
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**MATCHING:** Given segmentations, find the maximal matching.



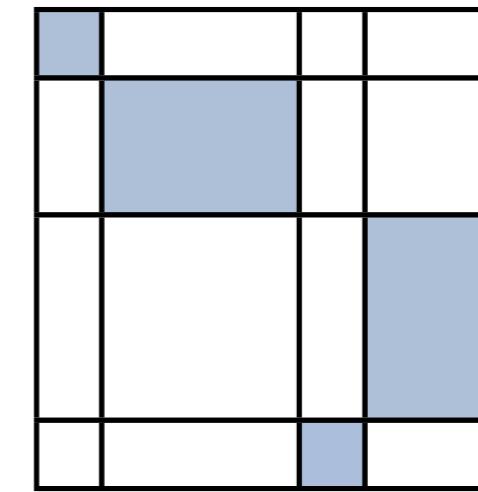
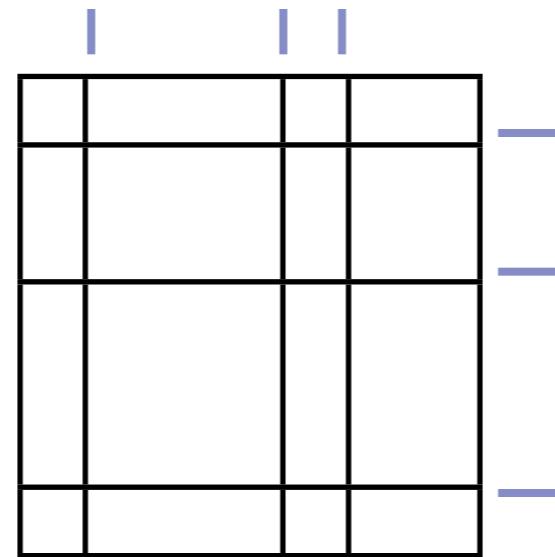
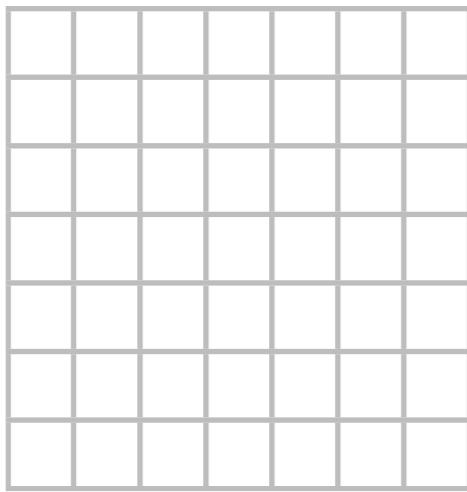
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# Related Problems are Polynomial

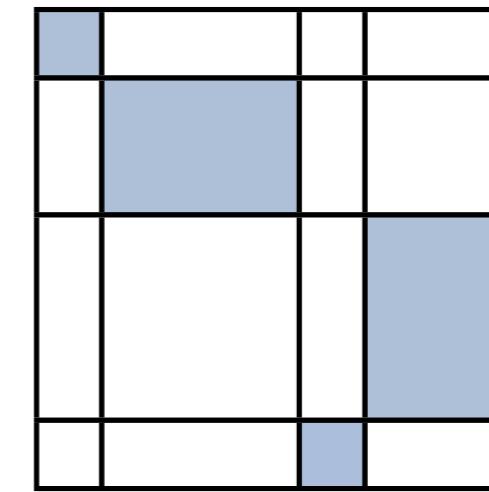
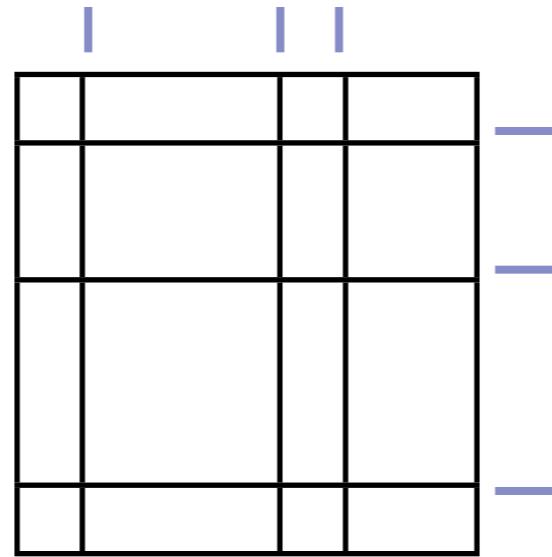
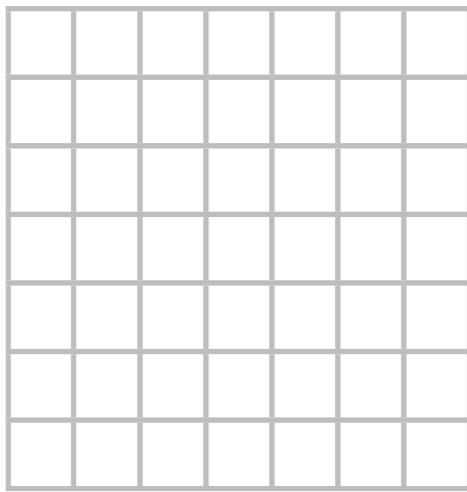
**MATCHING:** Given segmentations, find the maximal matching.



**PARTITIONING:** Given phrase weights, find the max segmentation.

# Related Problems are Polynomial

**MATCHING:** Given segmentations, find the maximal matching.

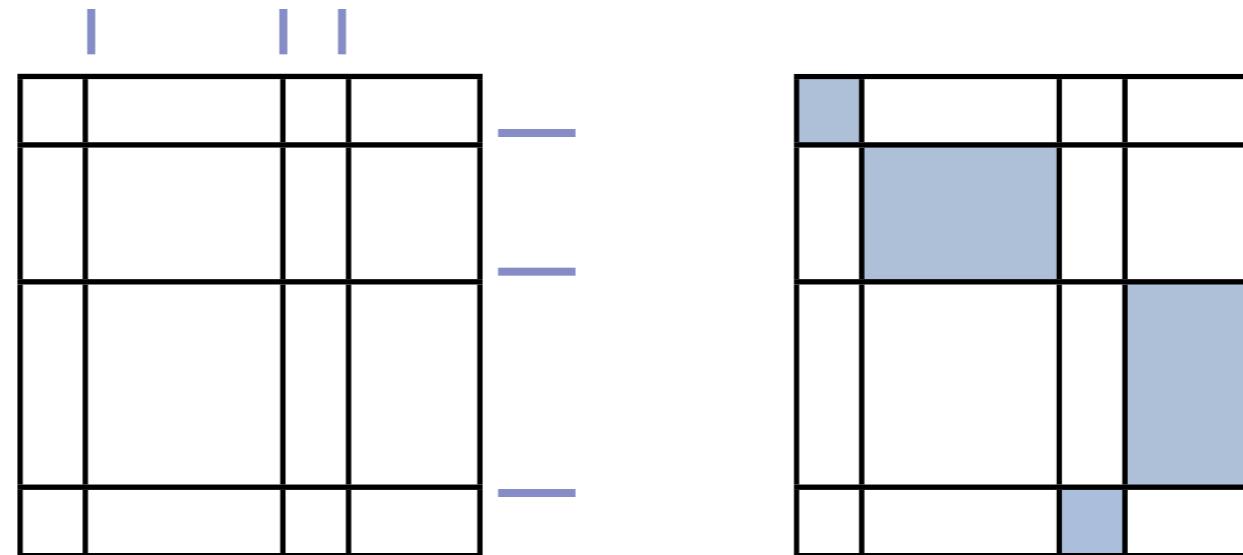
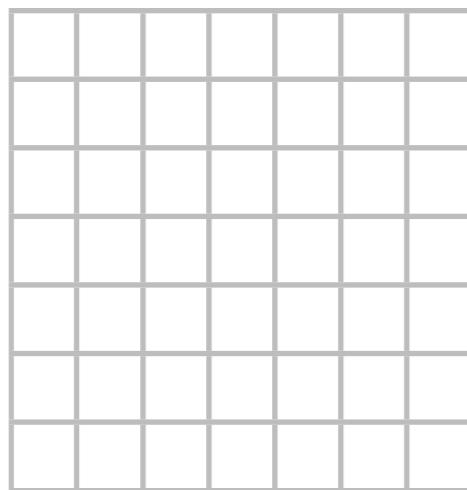


**PARTITIONING:** Given phrase weights, find the max segmentation.

*Japan to freeze aid to Russia .*

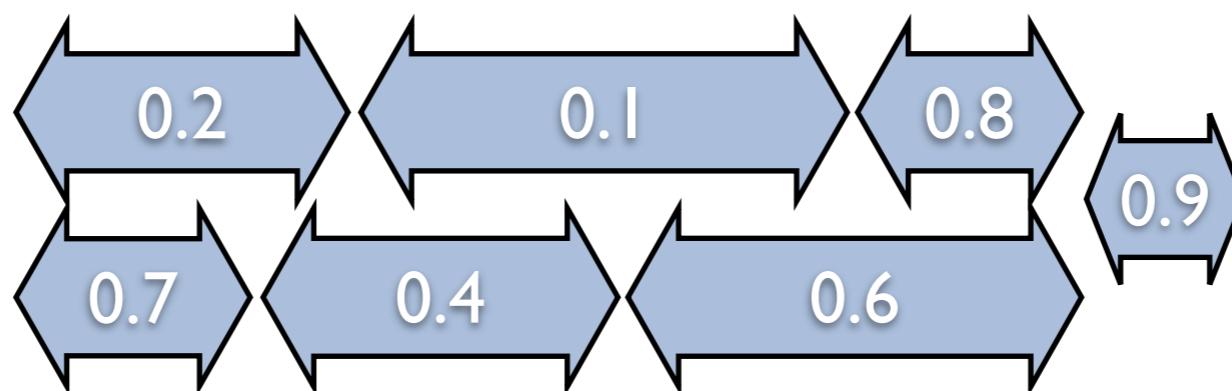
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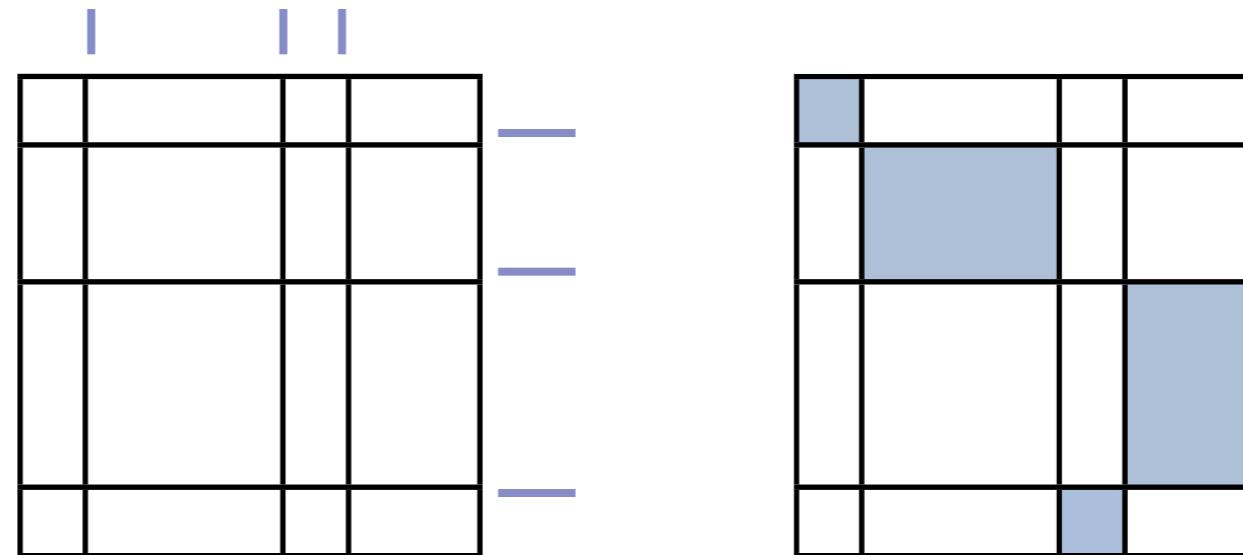
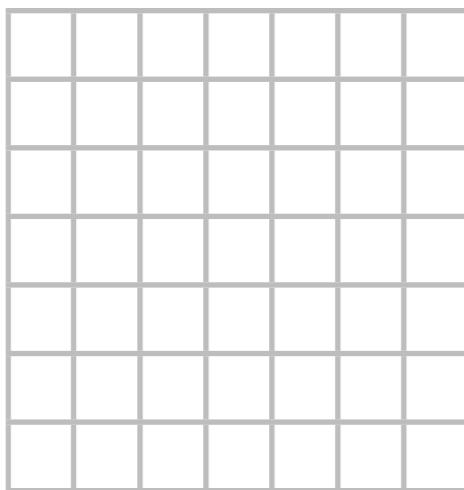
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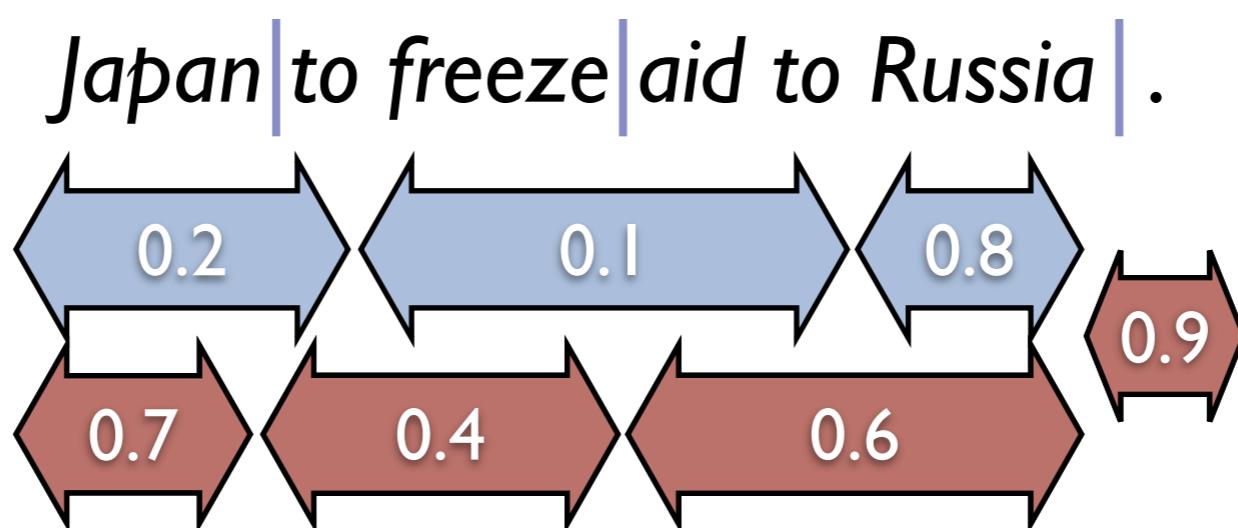


# Related Problems are Polynomial

**MATCHING:** Given segmentations, find the maximal matching.



**PARTITIONING:** Given phrase weights, find the max segmentation.



# The Phrase Alignment Problems

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*Given a sentence pair and scores for all phrase pairs:*

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- PHRASE OPTIMIZATION: Find the highest scoring phrase alignment.

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*Given a sentence pair and scores for all phrase pairs:*

- PHRASE OPTIMIZATION: Find the highest scoring phrase alignment.
- PHRASE DECISION: Determine if there is a phrase alignment with score  $\geq t$ .

# Reducing 3-SAT to PHRASE DECISION

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## *Anatomy of a reduction*

- Choose an old problem that is known to be NP-hard.
- Show that we can solve that old problem easily if we can solve our new problem.
- Conclude that if the new problem were in P, the old problem would be too (which it's not, we think).

# Reducing 3-SAT to PHRASE DECISION

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$$v_1 \vee v_2 \vee v_3$$

$$\bar{v}_1 \vee v_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

# Reducing 3-SAT to PHRASE DECISION

$$v_1 \vee v_2 \vee v_3$$

$$\bar{v}_1 \vee v_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

Satisfying  
assignment:

$v_1$  is *true*

$v_2$  is *false*

$v_3$  is *false*

# Reducing 3-SAT to PHRASE DECISION

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$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$



# Reducing 3-SAT to PHRASE DECISION

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$v_1 \quad \bar{v}_1 \quad v_2 \quad \bar{v}_2 \quad v_3 \quad \bar{v}_3$

$v_1 \vee v_2 \vee v_3$

$\bar{v}_1 \vee v_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee v_3$

# Reducing 3-SAT to PHRASE DECISION

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$v_1 \vee v_2 \vee v_3$   
 $\bar{v}_1 \vee v_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee v_3$

$v_1 \quad \bar{v}_1 \quad v_2 \quad \bar{v}_2 \quad v_3 \quad \bar{v}_3$


# Reducing 3-SAT to PHRASE DECISION

$v_1 \quad \bar{v}_1 \quad v_2 \quad \bar{v}_2 \quad v_3 \quad \bar{v}_3$

$v_1 \vee v_2 \vee v_3$

$\bar{v}_1 \vee v_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee v_3$




# Reducing 3-SAT to PHRASE DECISION

$$v_1 \vee v_2 \vee v_3$$

$$\bar{v}_1 \vee v_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

$$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$$



# Reducing 3-SAT to PHRASE DECISION

$$v_1 \vee v_2 \vee v_3$$

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$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

$$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$$



# Reducing 3-SAT to PHRASE DECISION

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$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

$$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$$



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$$\bar{v}_1 \vee v_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

$$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$$



# Reducing 3-SAT to PHRASE DECISION

$$v_1 \vee v_2 \vee v_3$$

$$\bar{v}_1 \vee v_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$$

$$\bar{v}_1 \vee \bar{v}_2 \vee v_3$$

$$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$$

A grid diagram consisting of 10 columns and 6 rows. The first column features a thick black vertical border on its left side. Four blue horizontal bars are positioned across the grid: one bar spans the width of columns 1 and 2 in the top row, and three bars span the widths of columns 3-4, 5-6, and 7-8 respectively in the second row.

# Reducing 3-SAT to PHRASE DECISION

$v_1 \vee v_2 \vee v_3$   
 $\bar{v}_1 \vee v_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee v_3$

$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$

	—				—	—			—	—	
$v_1 \vee v_2 \vee v_3$					—	—					
$\bar{v}_1 \vee v_2 \vee \bar{v}_3$	—	—	—	—	—	—			—	—	
$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$	—	—	—			—	—		—	—	
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$	—	—	—			—	—	—	—		

# Reducing 3-SAT to PHRASE DECISION

$v_1 \vee v_2 \vee v_3$   
 $\bar{v}_1 \vee v_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee v_3$

	$v_1$	$\bar{v}_1$	$\bar{v}_1$	$\bar{v}_1$	$v_2$	$v_2$	$\bar{v}_2$	$\bar{v}_2$	$v_3$	$v_3$	$\bar{v}_3$	$\bar{v}_3$
$v_1 \vee v_2 \vee v_3$		—				—	—			—	—	
$\bar{v}_1 \vee v_2 \vee \bar{v}_3$			—	—	—	—	—				—	—
$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$			—	—	—			—	—		—	—
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$			—	—	—			—	—	—	—	

# Reducing 3-SAT to PHRASE DECISION

$v_1 \vee v_2 \vee v_3$   
 $\bar{v}_1 \vee v_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$   
 $\bar{v}_1 \vee \bar{v}_2 \vee v_3$

$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$

	—				—	—			—	—	
$v_1 \vee v_2 \vee v_3$					—	—					
$\bar{v}_1 \vee v_2 \vee \bar{v}_3$	—	—	—	—	—	—			—	—	
$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$	—	—	—			—	—		—	—	
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$	—	—	—			—	—	—	—		



# Reducing 3-SAT to PHRASE DECISION

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$v_1 \vee v_2 \vee v_3$

$\bar{v}_1 \vee v_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$

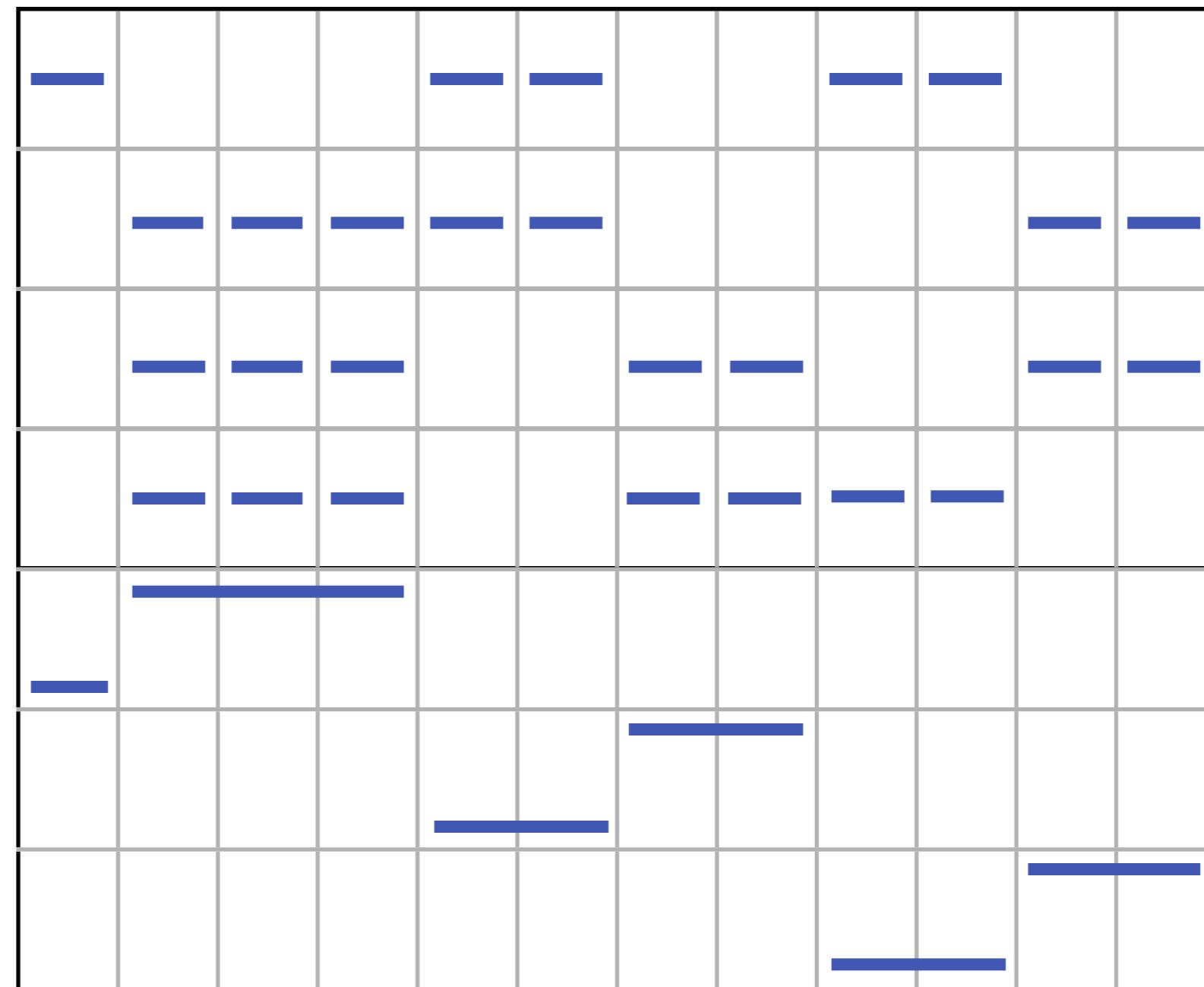
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$

assign( $v_1$ )

assign( $v_2$ )

assign( $v_3$ )

$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$



# Reducing 3-SAT to PHRASE DECISION

 $v_1 \vee v_2 \vee v_3$ 
 $\bar{v}_1 \vee v_2 \vee \bar{v}_3$ 
 $\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$ 
 $\bar{v}_1 \vee \bar{v}_2 \vee v_3$ 
 $\text{assign}(v_1)$ 
 $\text{assign}(v_2)$ 
 $\text{assign}(v_3)$ 

	$v_1$	$\bar{v}_1$	$\bar{v}_1$	$\bar{v}_1$	$v_2$	$v_2$	$\bar{v}_2$	$\bar{v}_2$	$v_3$	$v_3$	$\bar{v}_3$	$\bar{v}_3$
--	-------	-------------	-------------	-------------	-------	-------	-------------	-------------	-------	-------	-------------	-------------

$v_1 \vee v_2 \vee v_3$	—				—	—			—	—		
$\bar{v}_1 \vee v_2 \vee \bar{v}_3$		—	—	—	—	—				—	—	—
$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$		—	—	—			—	—			—	—
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$		—	—	—			—	—	—	—	—	
$\text{assign}(v_1)$	—	—	—	—								
$\text{assign}(v_2)$					—	—	—	—				
$\text{assign}(v_3)$									—	—	—	—

# Reducing 3-SAT to PHRASE DECISION

$v_1 \vee v_2 \vee v_3$

$\bar{v}_1 \vee v_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$

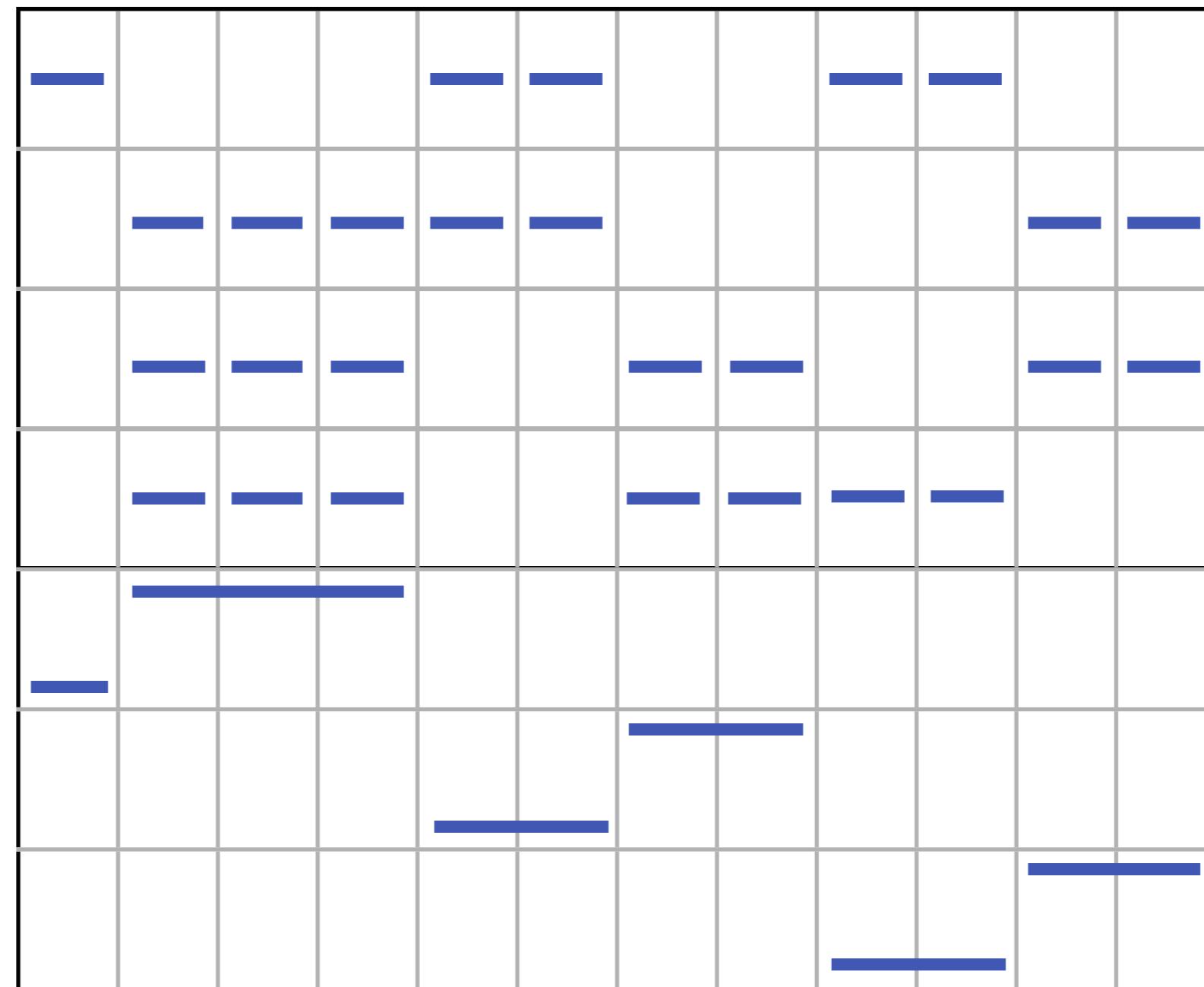
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$

assign( $v_1$ )

assign( $v_2$ )

assign( $v_3$ )

$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$



# Reducing 3-SAT to PHRASE DECISION

$v_1 \vee v_2 \vee v_3$

$\bar{v}_1 \vee v_2 \vee \bar{v}_3$

$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$

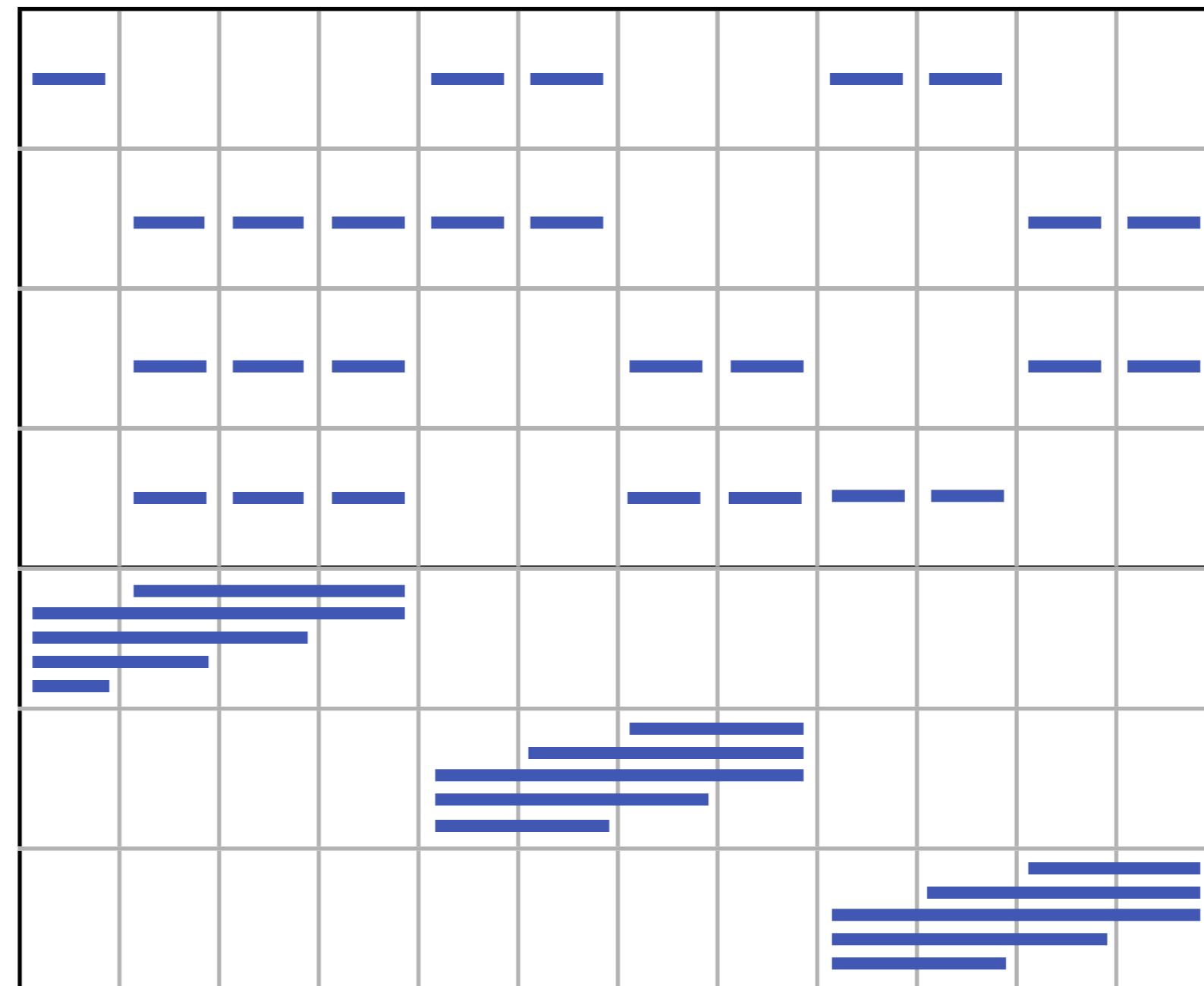
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$

assign( $v_1$ )

assign( $v_2$ )

assign( $v_3$ )

$v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$



# Reducing 3-SAT to PHRASE DECISION

 $v_1 \vee v_2 \vee v_3$ 
 $\bar{v}_1 \vee v_2 \vee \bar{v}_3$ 
 $\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$ 
 $\bar{v}_1 \vee \bar{v}_2 \vee v_3$ 
 $\text{assign}(v_1)$ 
 $\text{assign}(v_2)$ 
 $\text{assign}(v_3)$ 
 $v_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad \bar{v}_1 \quad v_2 \quad v_2 \quad \bar{v}_2 \quad \bar{v}_2 \quad v_3 \quad v_3 \quad \bar{v}_3 \quad \bar{v}_3$ 

	—			— —			— —				
$v_1 \vee v_2 \vee v_3$											
$\bar{v}_1 \vee v_2 \vee \bar{v}_3$		— — —	— — —						— —	— —	
$\bar{v}_1 \vee \bar{v}_2 \vee \bar{v}_3$		— — —			— —	— —			— —	— —	
$\bar{v}_1 \vee \bar{v}_2 \vee v_3$		— — —			— —	— —	— —				
$\text{assign}(v_1)$	— — —	— — —	— — —								
$\text{assign}(v_2)$				— — —	— — —	— — —	— — —				
$\text{assign}(v_3)$								— — —	— — —	— — —	



# Reducing 3-SAT to PHRASE DECISION



# Reducing 3-SAT to PHRASE DECISION



# Reducing 3-SAT to PHRASE DECISION



# Reducing 3-SAT to PHRASE DECISION

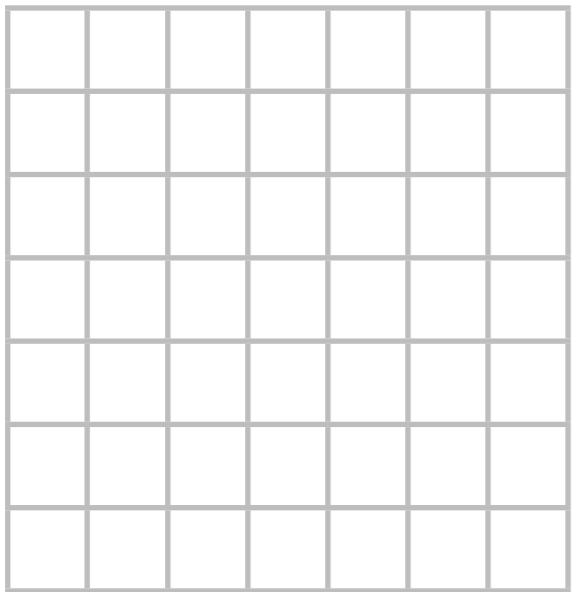


# Taking Expectations is #P-hard

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# Taking Expectations is #P-hard

Weighted grid:



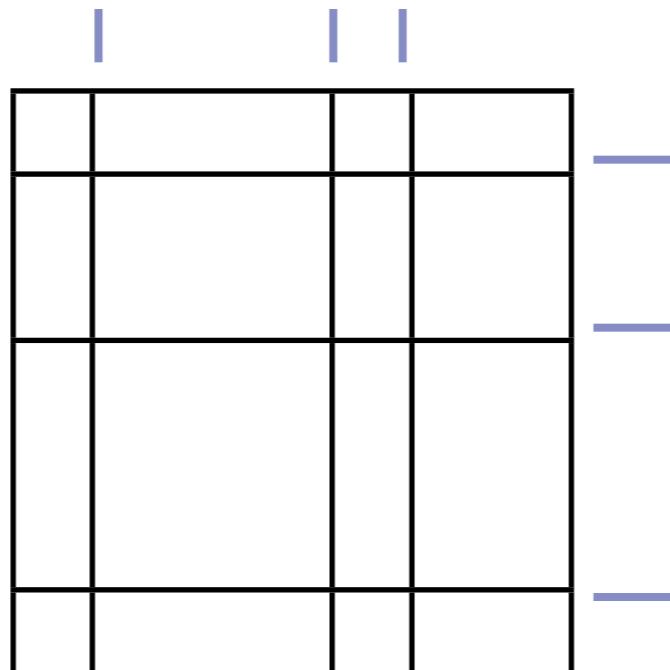
# Taking Expectations is #P-hard

Weighted grid:



+

Segmentation:



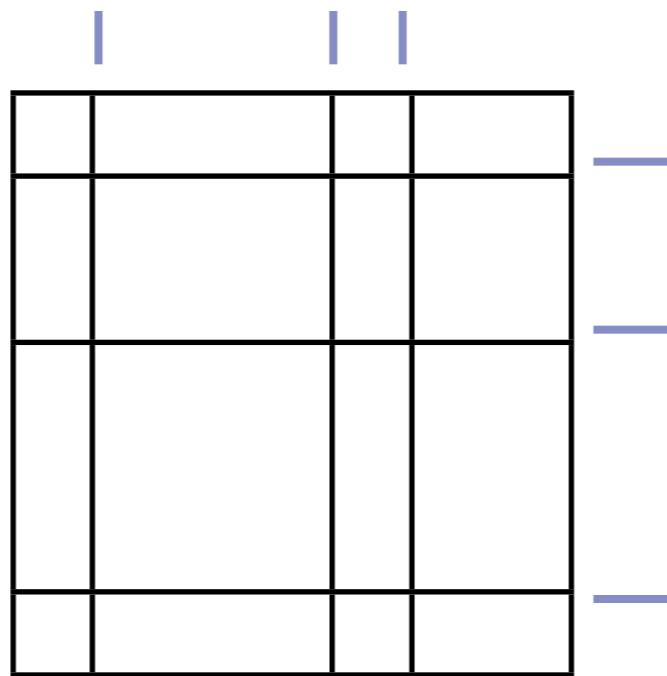
# Taking Expectations is #P-hard

Weighted grid:

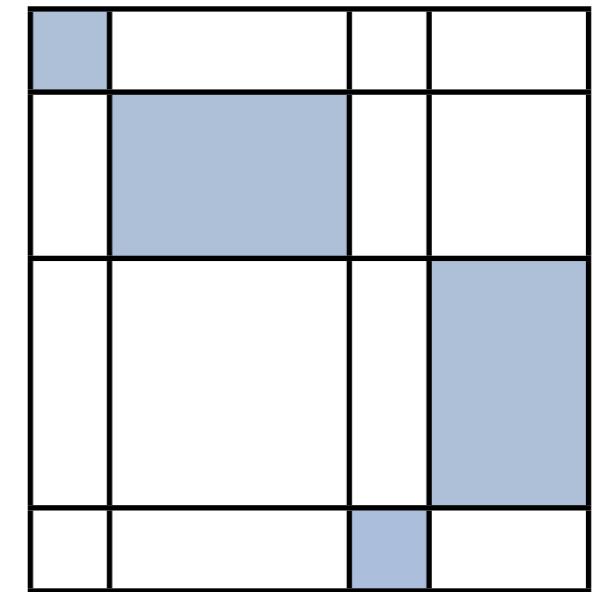


+

Segmentation:



Find Maximal:



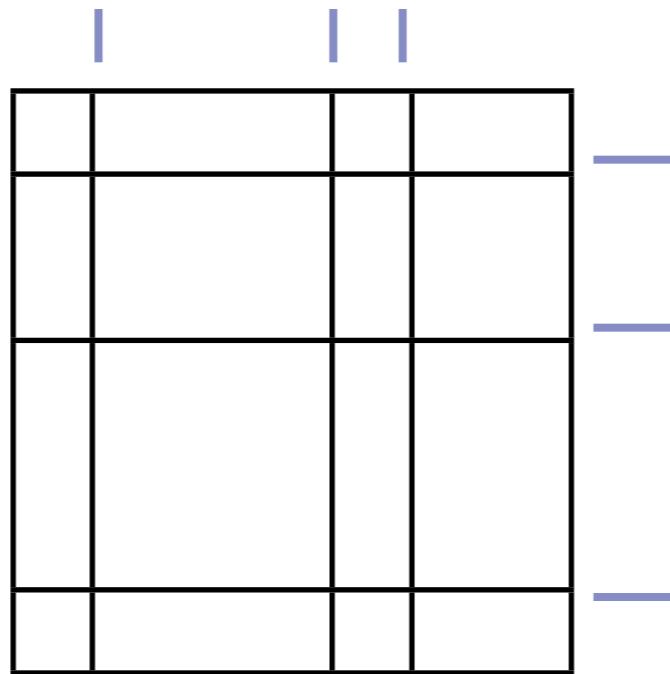
# Taking Expectations is #P-hard

Weighted grid:

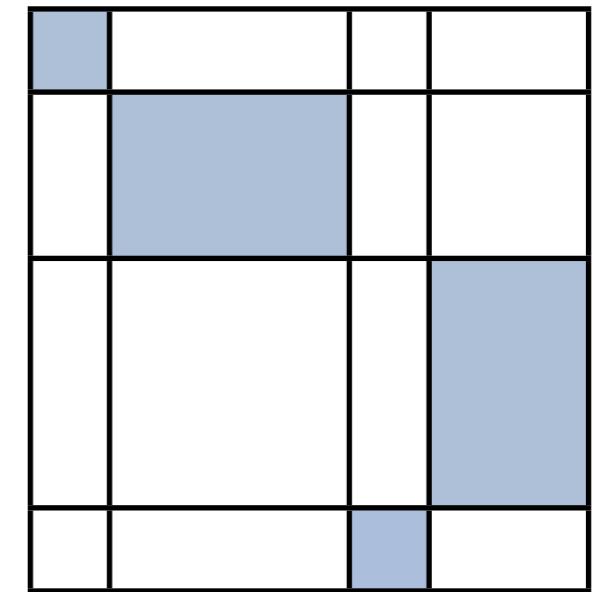


+

Segmentation:



Find Maximal:



Summing over matching is #P-hard

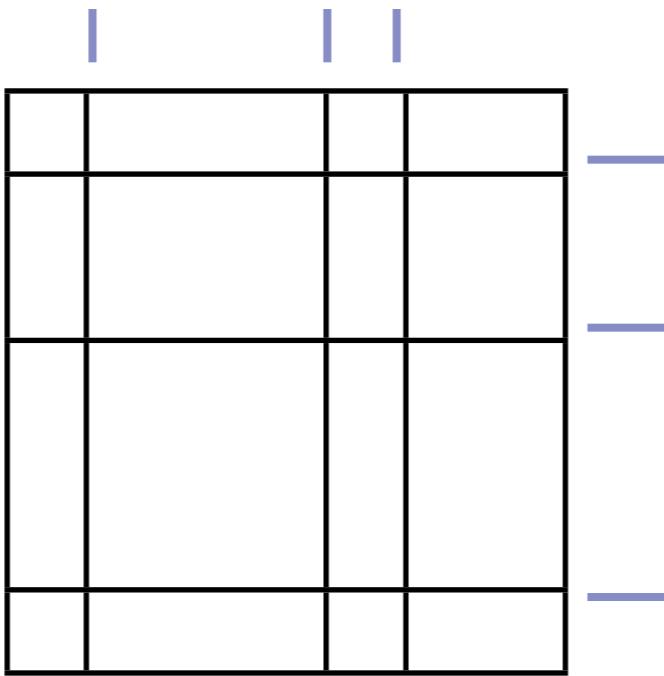
# Taking Expectations is #P-hard

Weighted grid:

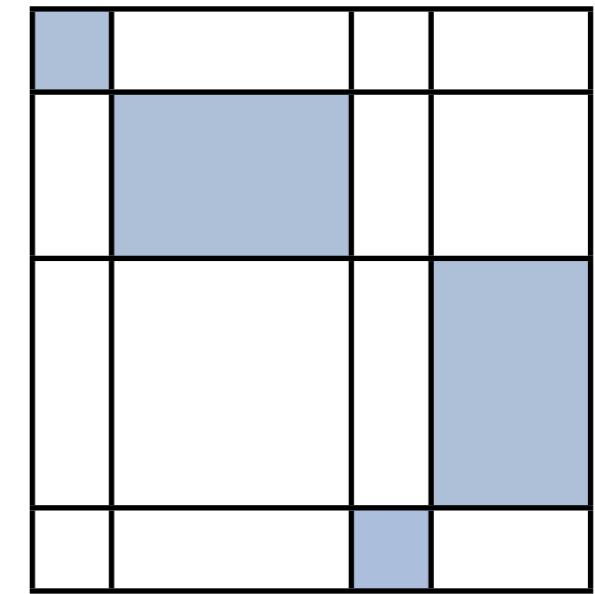


+

Segmentation:



Find Maximal:

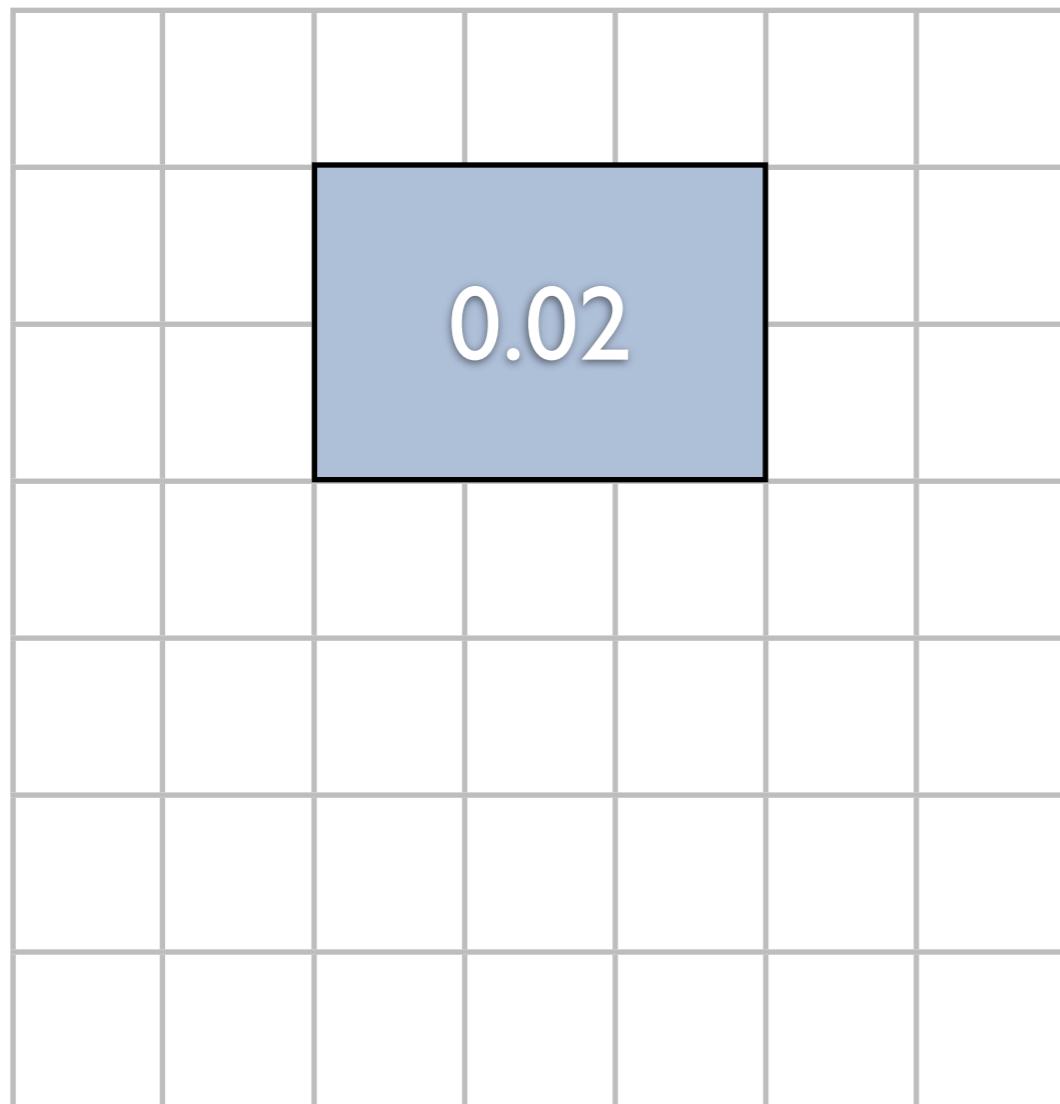


Summing over matching is #P-hard

Expectations of phrase alignments is at least as hard

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .



日本

冻结

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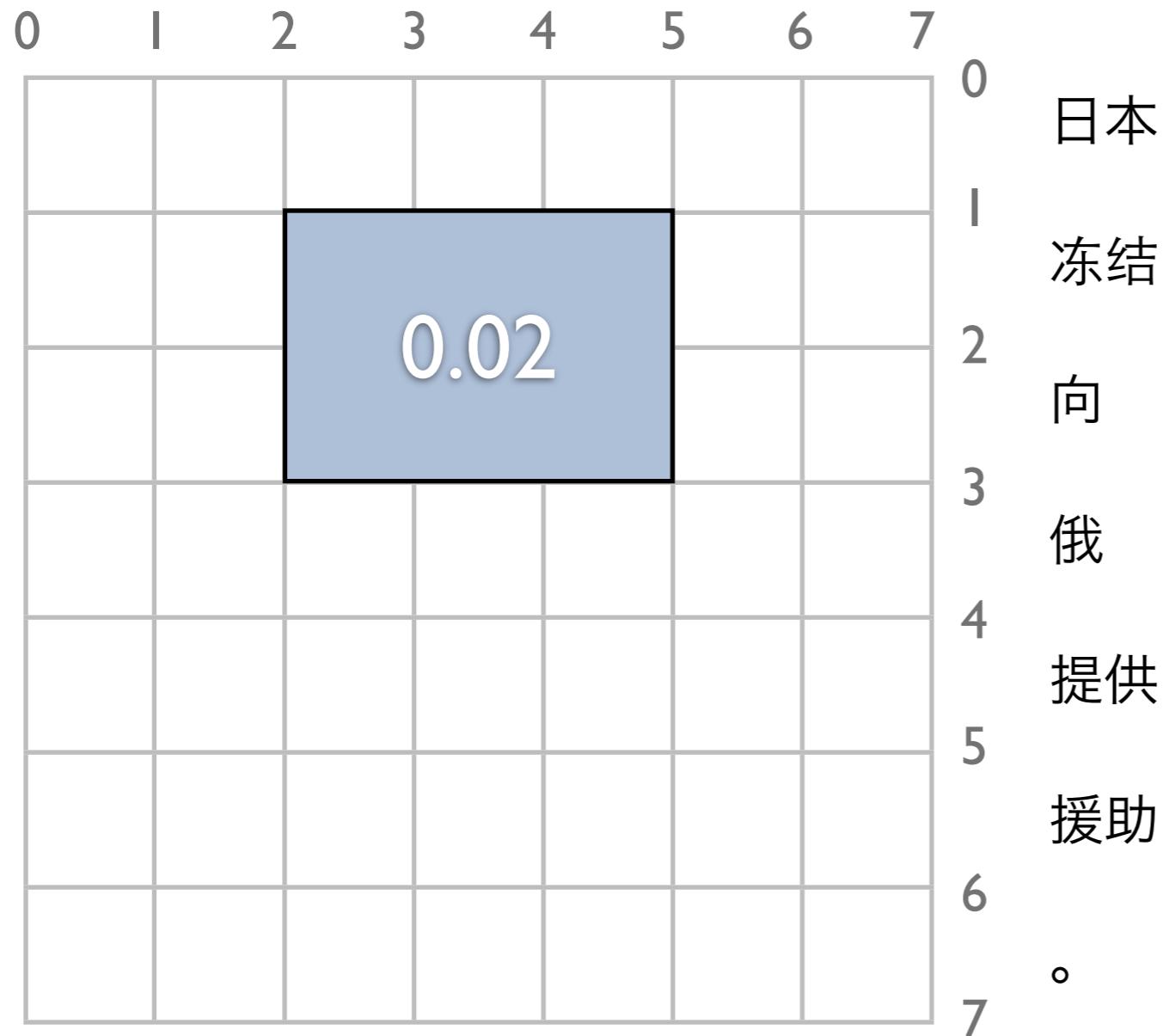
提供

援助

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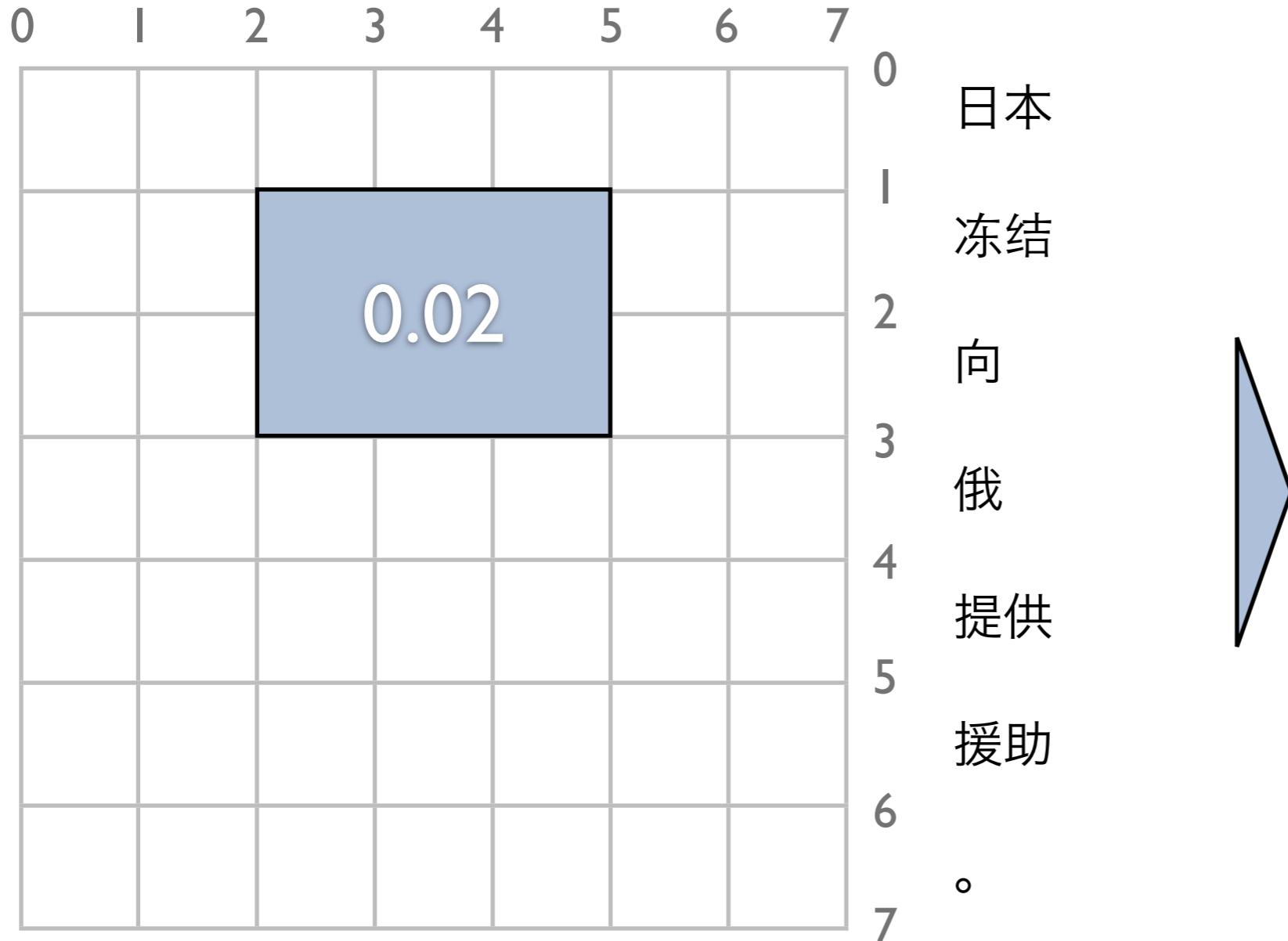
# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .



# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .

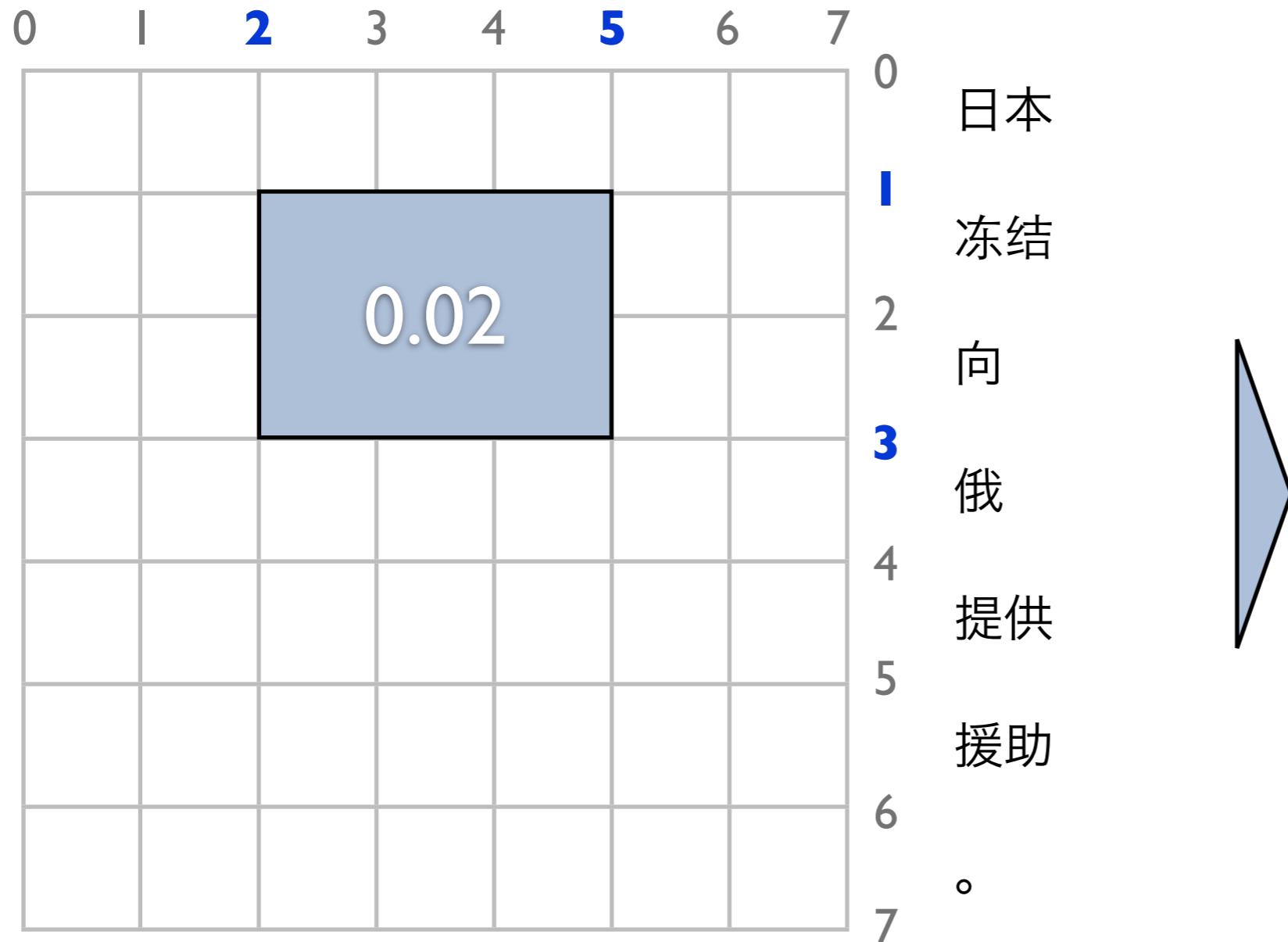


Constants:

$$w_{2,5,1,3} = \log 0.02$$

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .

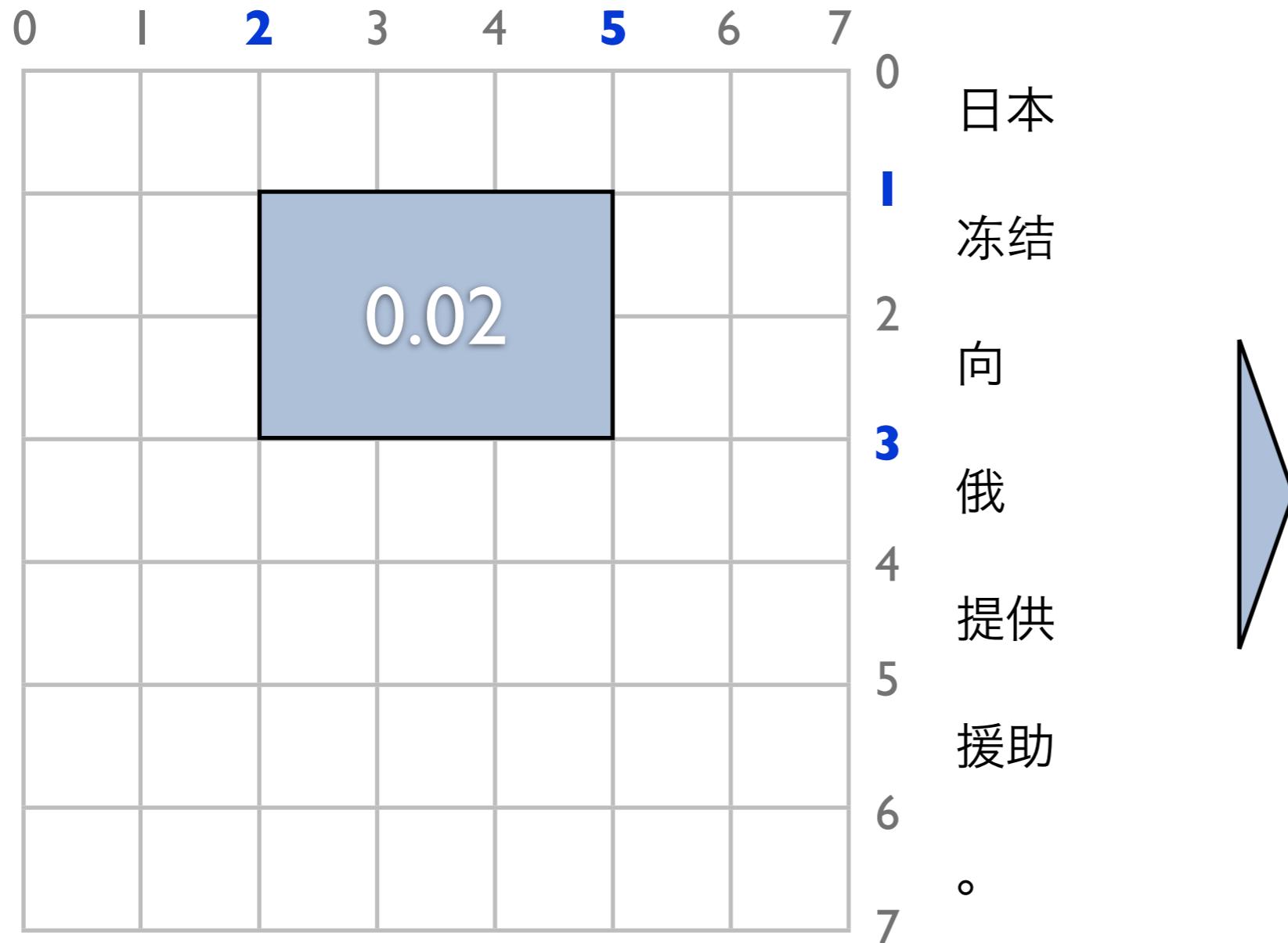


Constants:

$$w_{2,5,1,3} = \log 0.02$$

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .



Constants:

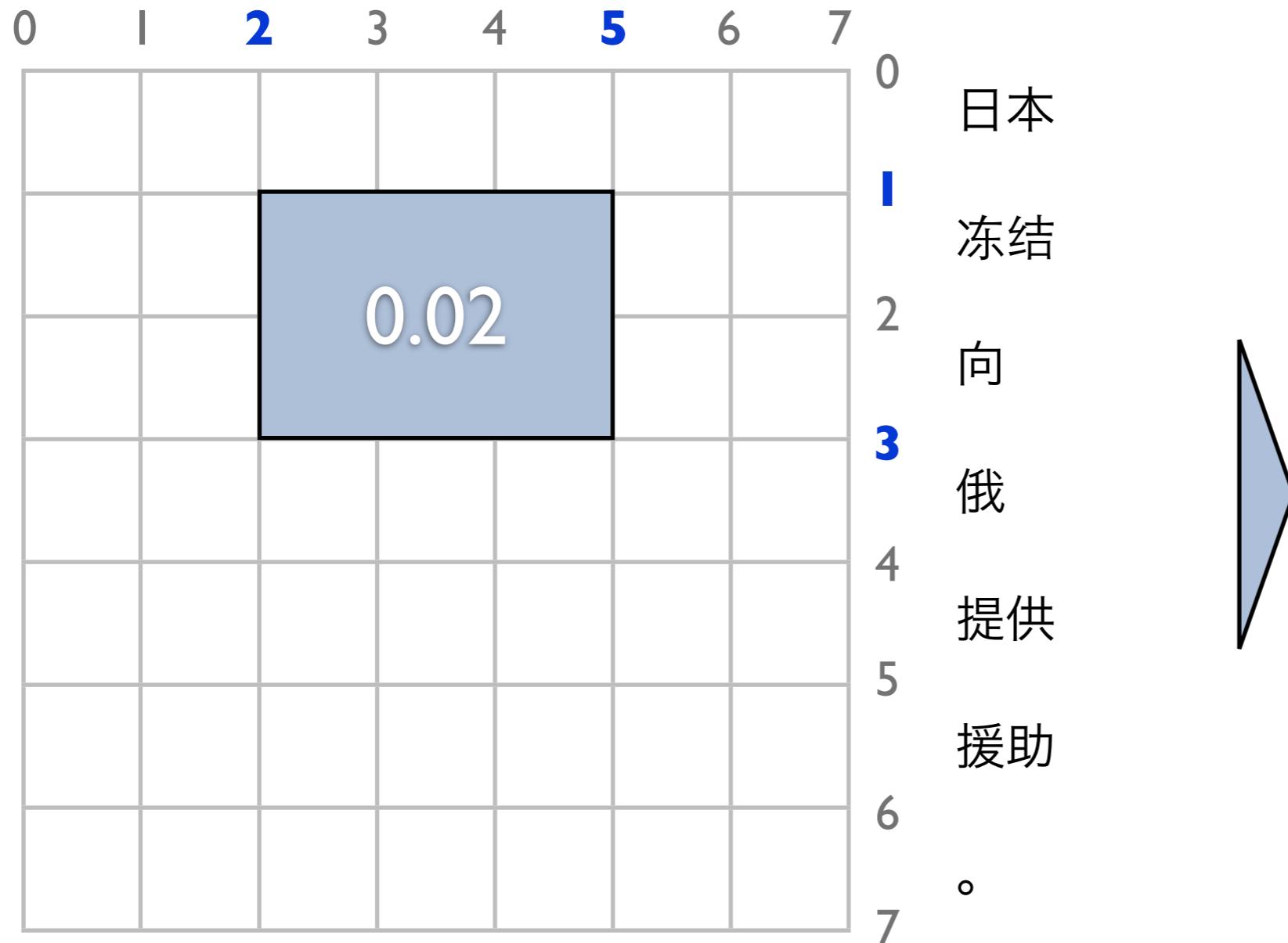
$$w_{2,5,1,3} = \log 0.02$$

Indicator Variables:

$$a_{2,5,1,3} = 1$$

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .



**Constants:**

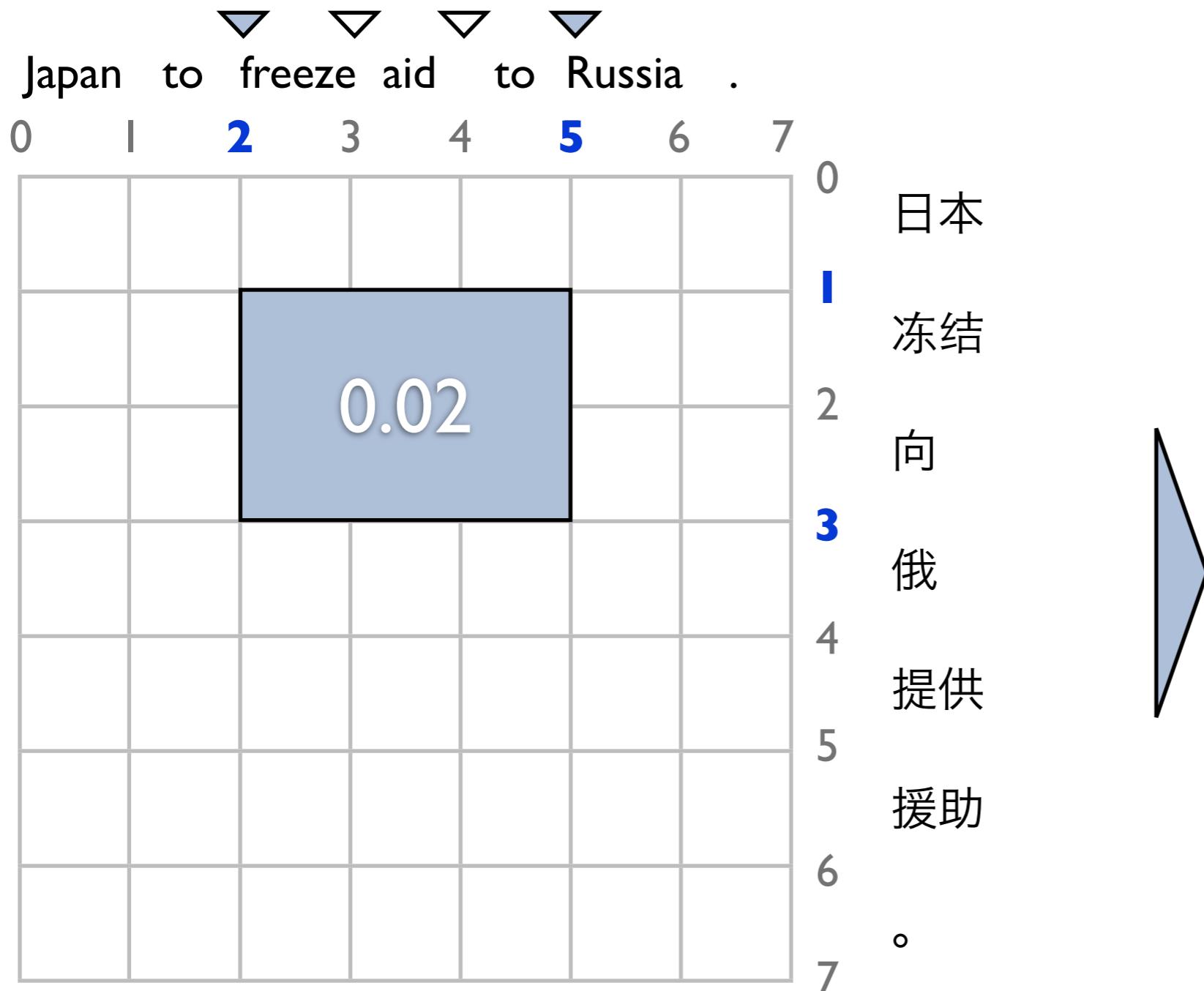
$$w_{2,5,1,3} = \log 0.02$$

**Indicator Variables:**

$$a_{2,5,1,3} = 1$$

$$e_{2,5} = 1$$

# Phrase Alignment as Integer Programming



**Constants:**

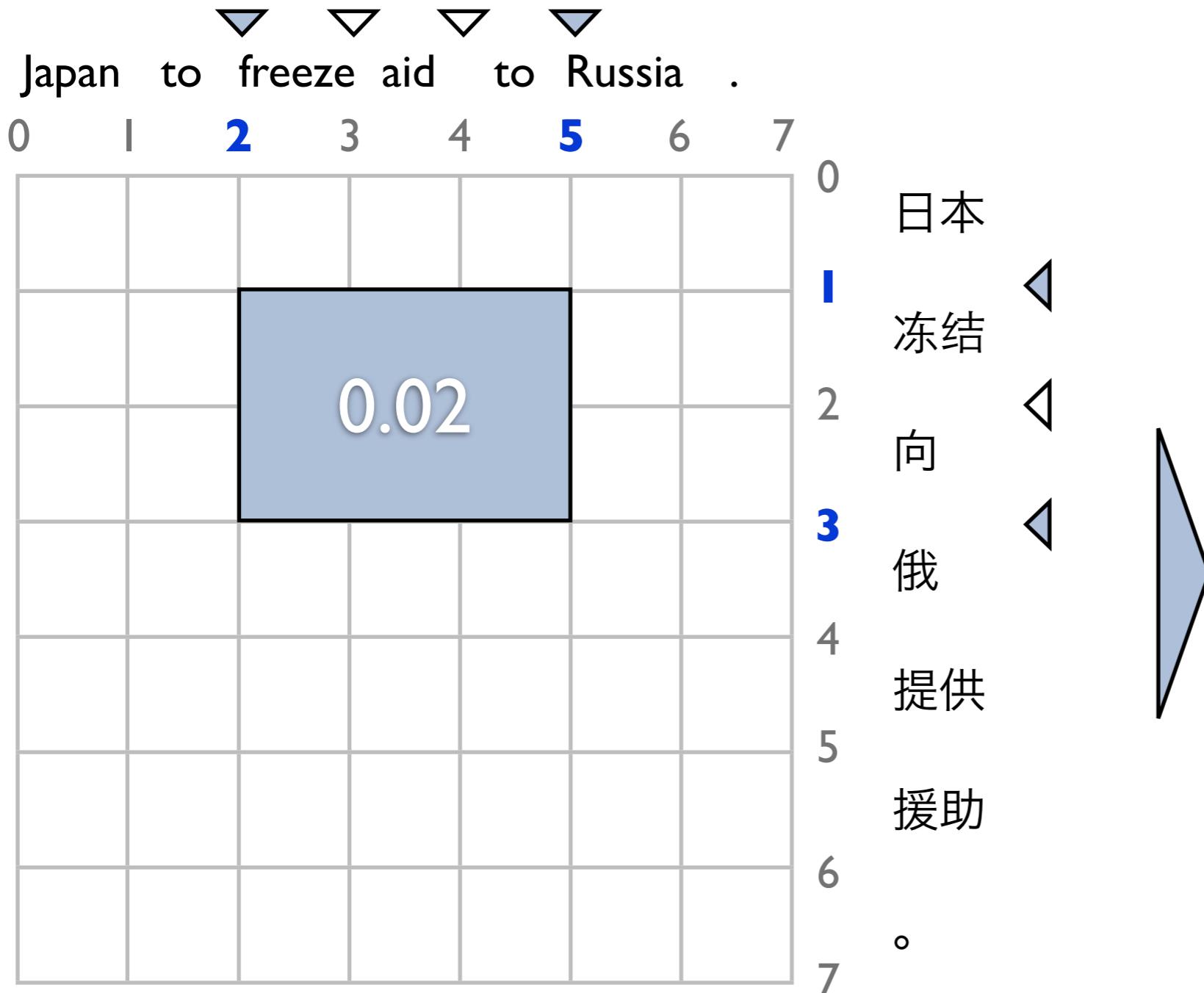
$$w_{2,5,1,3} = \log 0.02$$

**Indicator Variables:**

$$a_{2,5,1,3} = 1$$

$$e_{2,5} = 1$$

# Phrase Alignment as Integer Programming



Constants:

$$w_{2,5,1,3} = \log 0.02$$

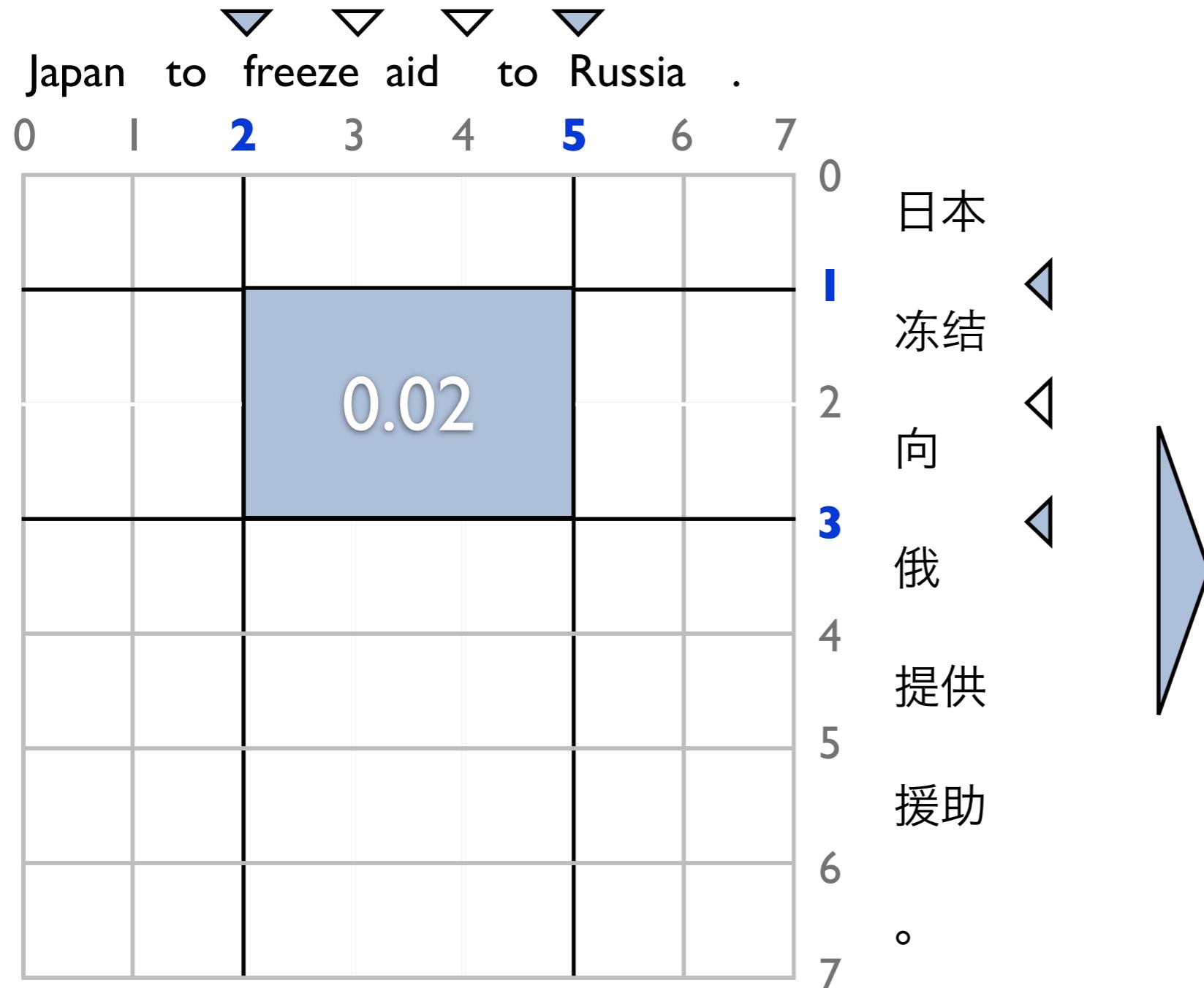
Indicator Variables:

$$a_{2,5,1,3} = 1$$

$$e_{2,5} = 1$$

$$f_{1,3} = 1$$

# Phrase Alignment as Integer Programming



**Constants:**

$$w_{2,5,1,3} = \log 0.02$$

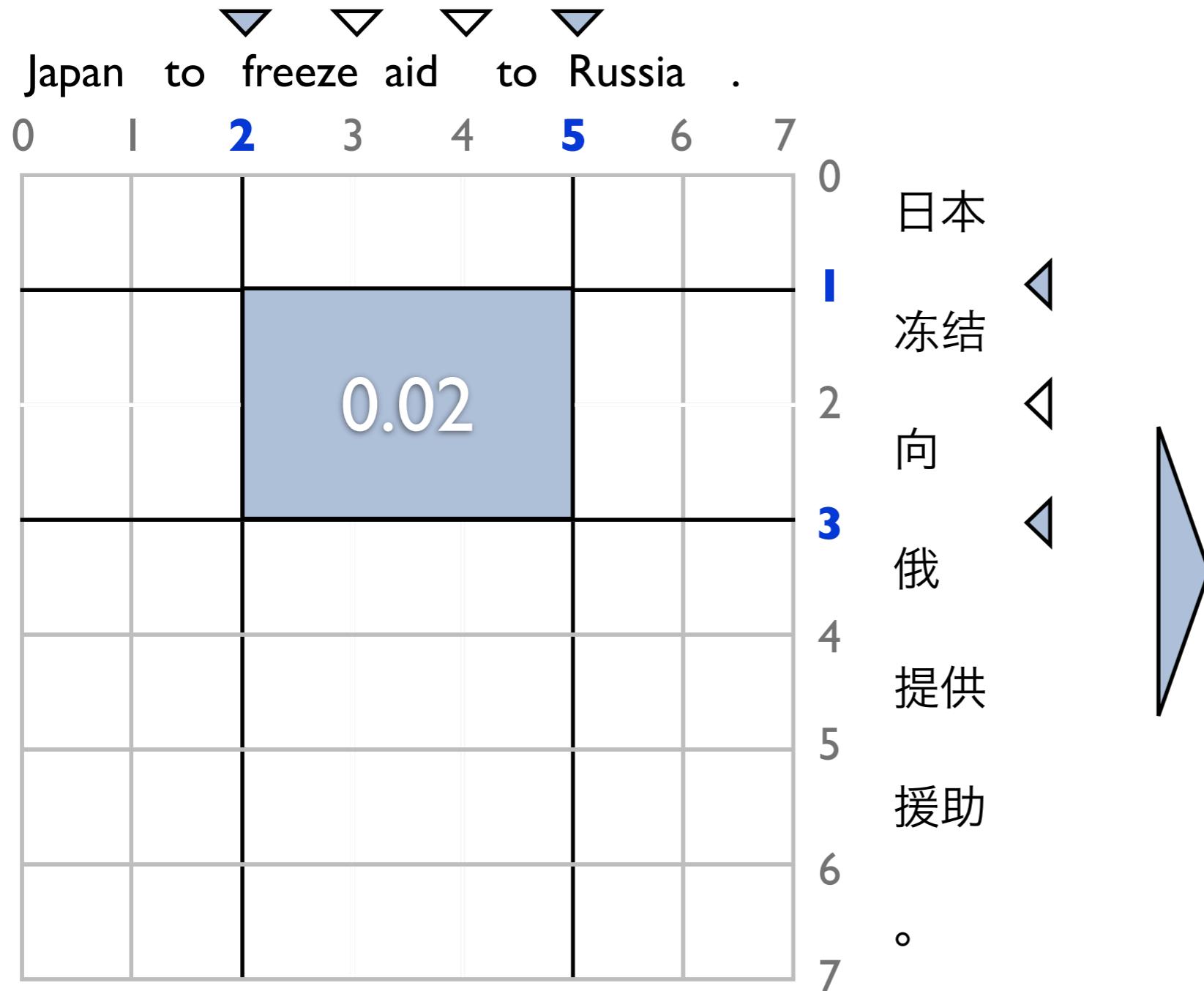
**Indicator Variables:**

$$a_{2,5,1,3} = 1$$

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# Phrase Alignment as Integer Programming



**Constants:**

$$w_{2,5,1,3} = \log 0.02$$

**Indicator Variables:**

$$a_{2,5,1,3} = 1$$

$$e_{2,5} = 1$$

$$f_{1,3} = 1$$

**Indicator Variables:**

**a:** phrase alignment

**e:** English segmentation

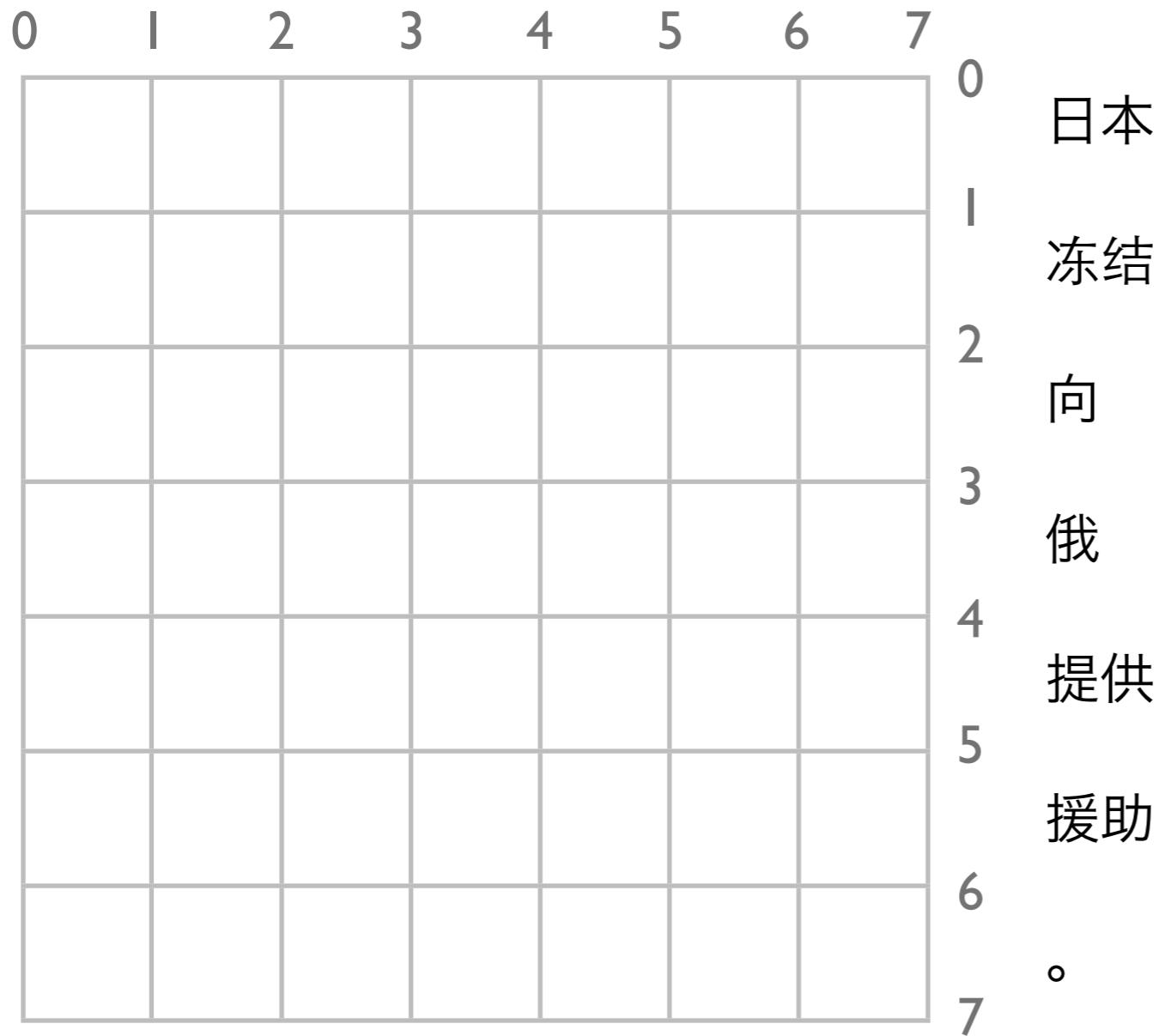
**f:** foreign segmentation

**Constants:**

**w:** weights

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .



*Indicator Variables:*

**a:** phrase alignment    **e:** English segmentation    **f:** foreign segmentation

*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .

0 | 2 3 4 5 6 7


日本  
冻结  
向  
俄  
提供  
援助  
。

$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$



*Indicator Variables:*

**a:** phrase alignment    **e:** English segmentation    **f:** foreign segmentation

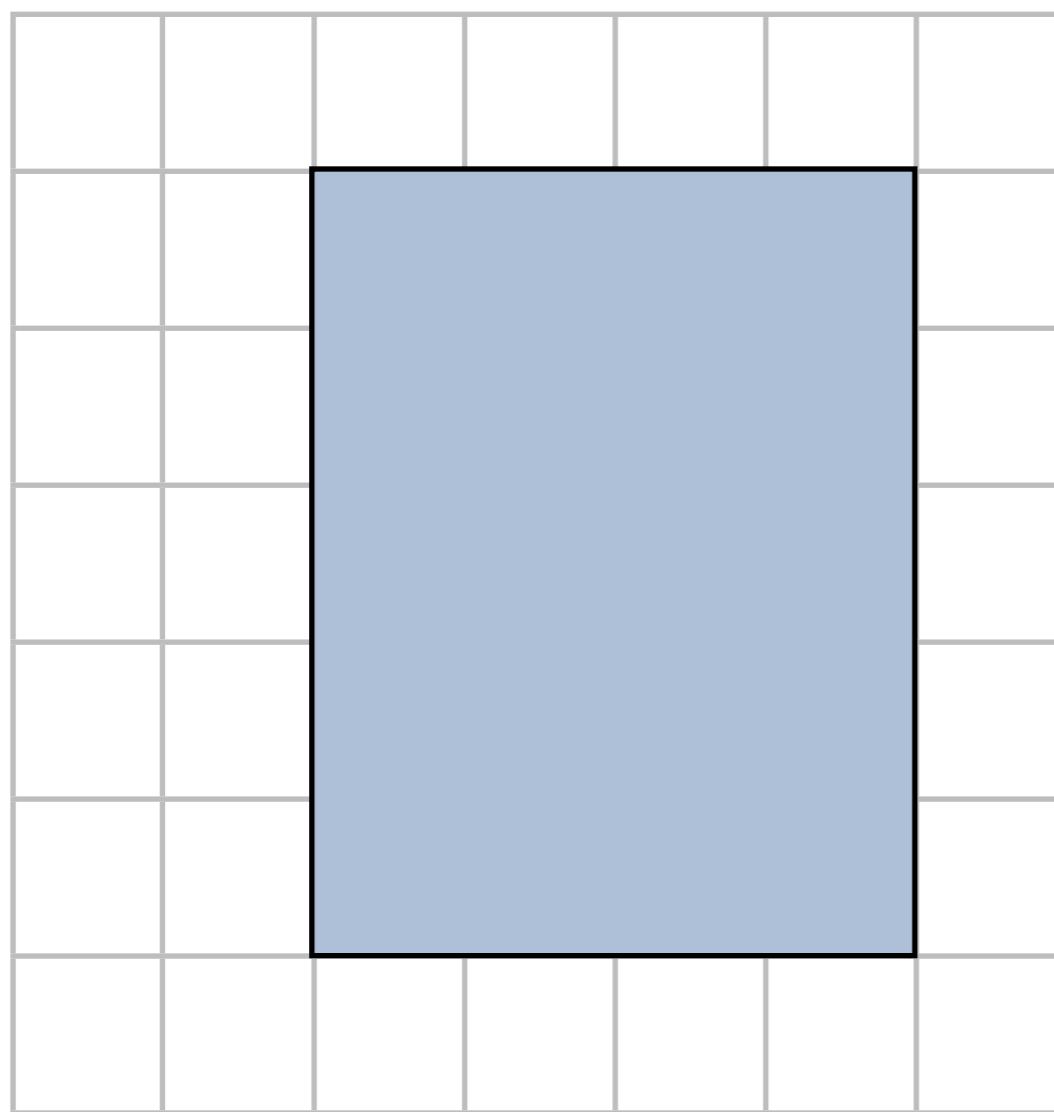
*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming

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$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$



*Indicator Variables:*

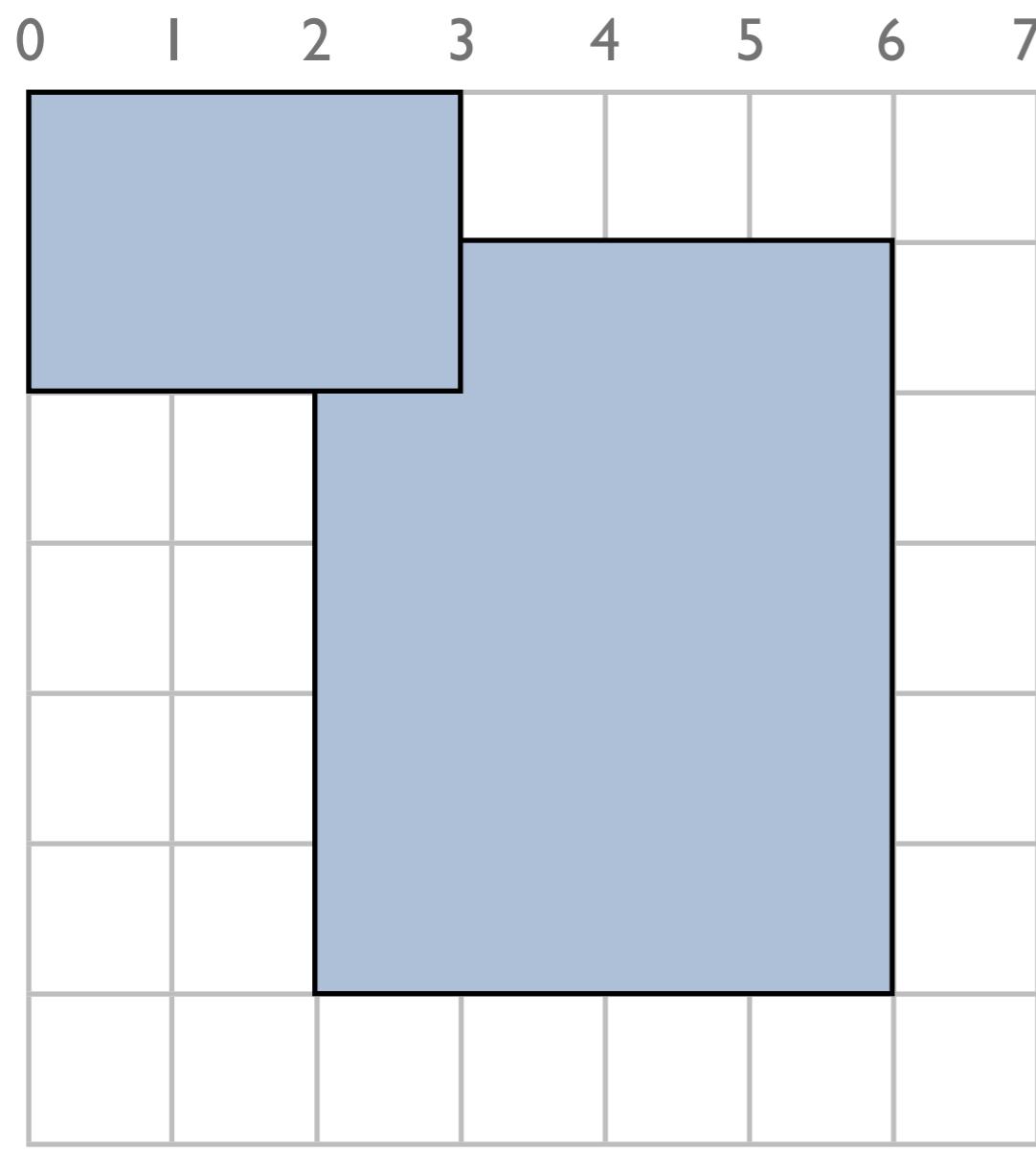
**a:** phrase alignment    **e:** English segmentation    **f:** foreign segmentation

*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .



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*Indicator Variables:*

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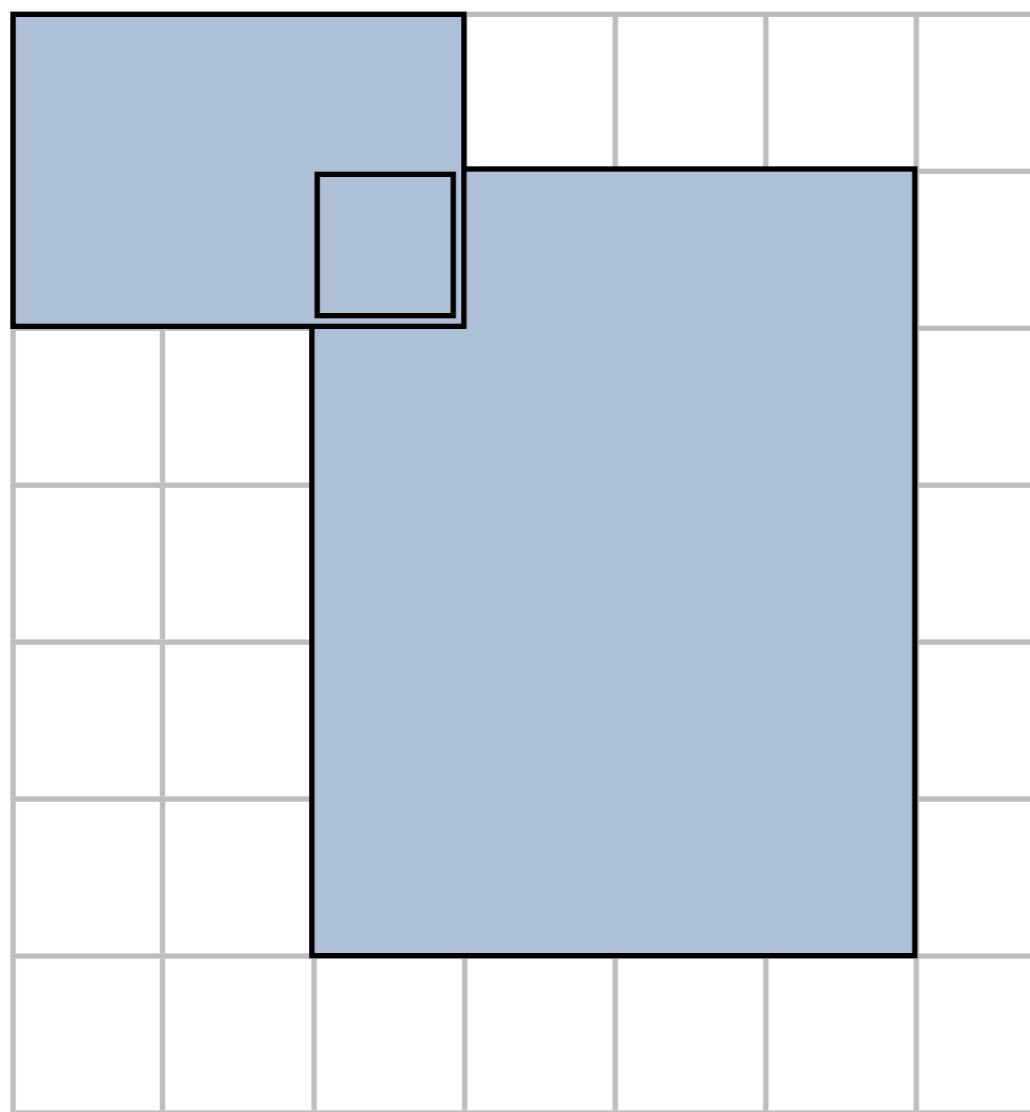
*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .

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$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$



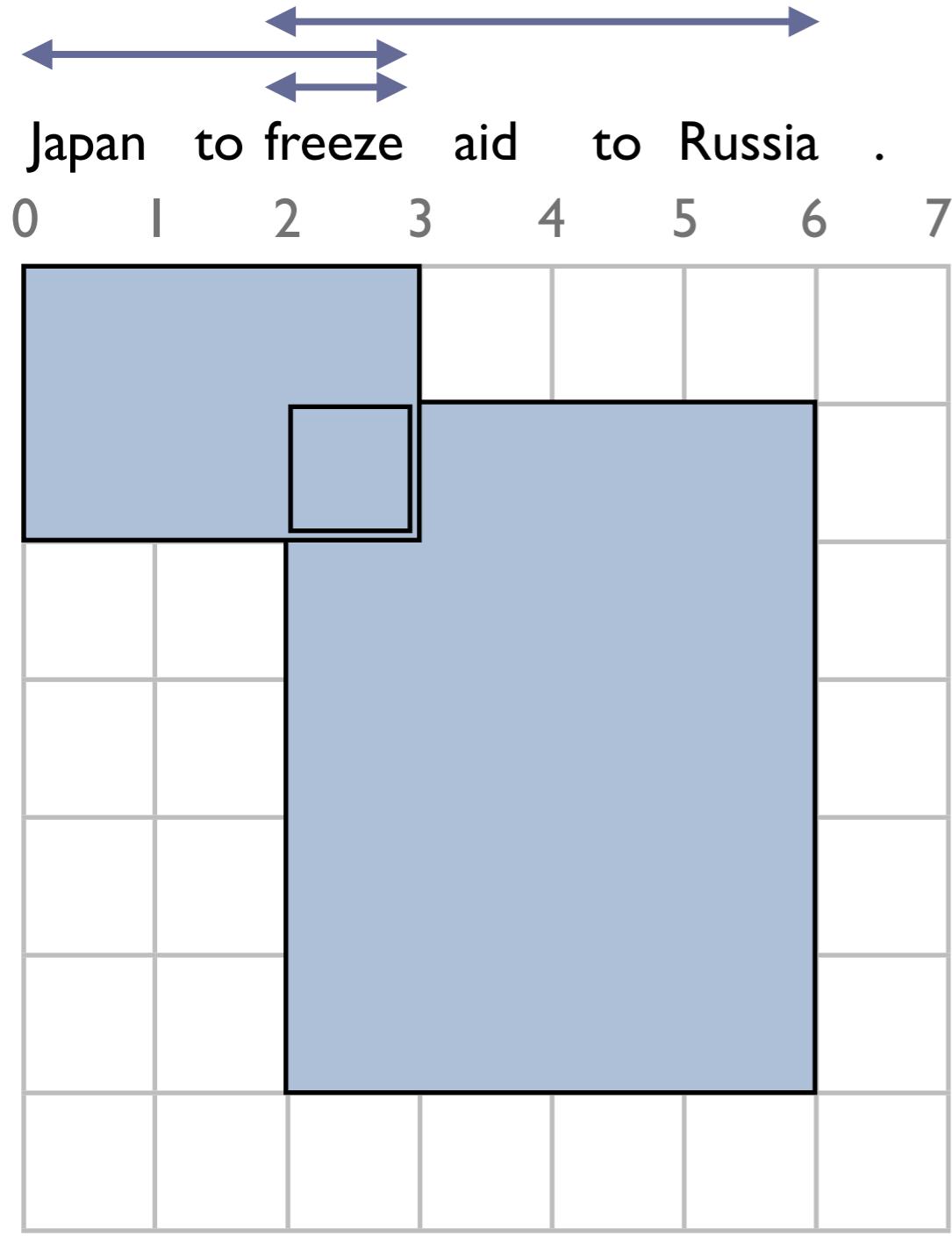
*Indicator Variables:*

**a:** phrase alignment    **e:** English segmentation    **f:** foreign segmentation

*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming



$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$

*Indicator Variables:*

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*Constants:*

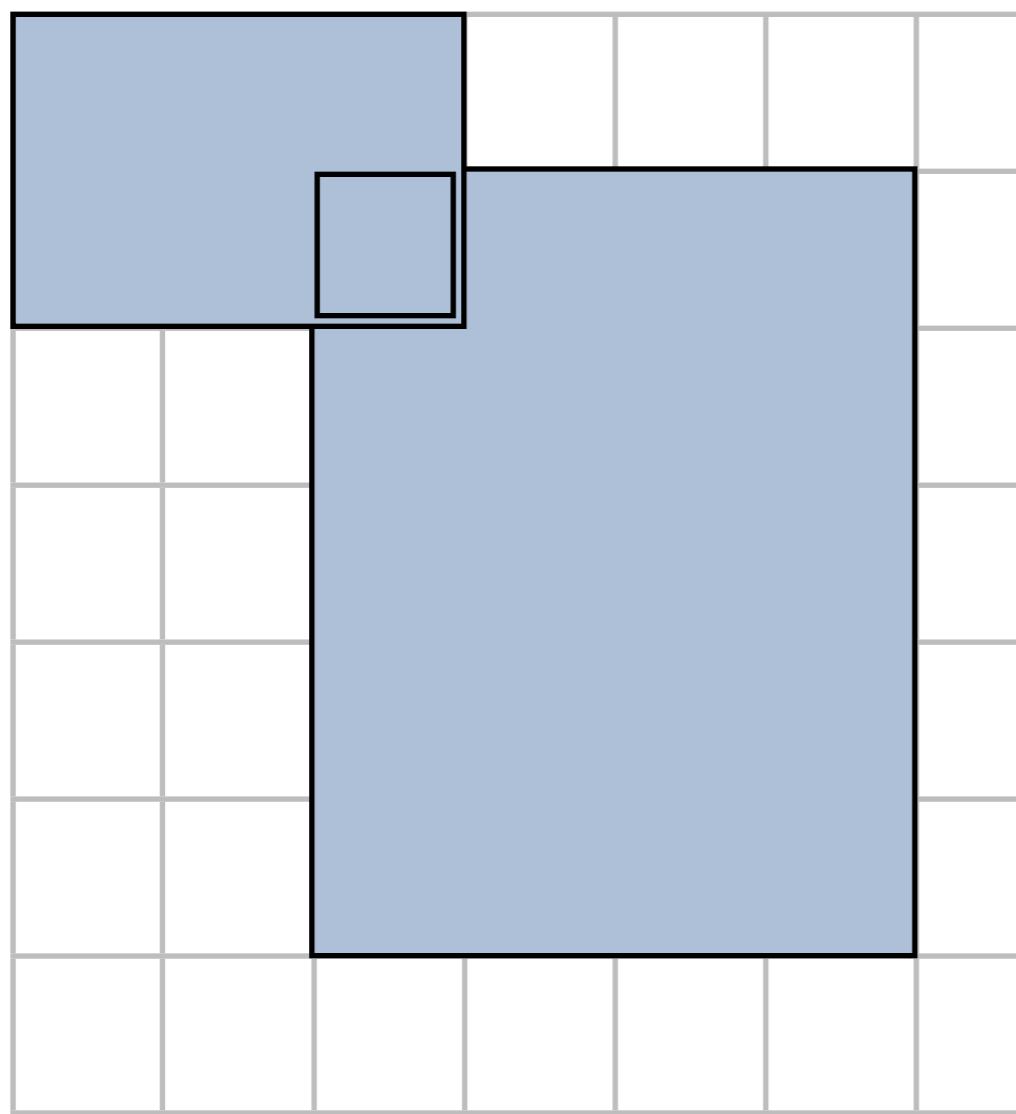
**w:** weights

# Phrase Alignment as Integer Programming



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$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$

$$\text{s.t. } \sum_{i,j: i < x \leq j} e_{i,j} = 1$$

*Indicator Variables:*

**a**: phrase alignment

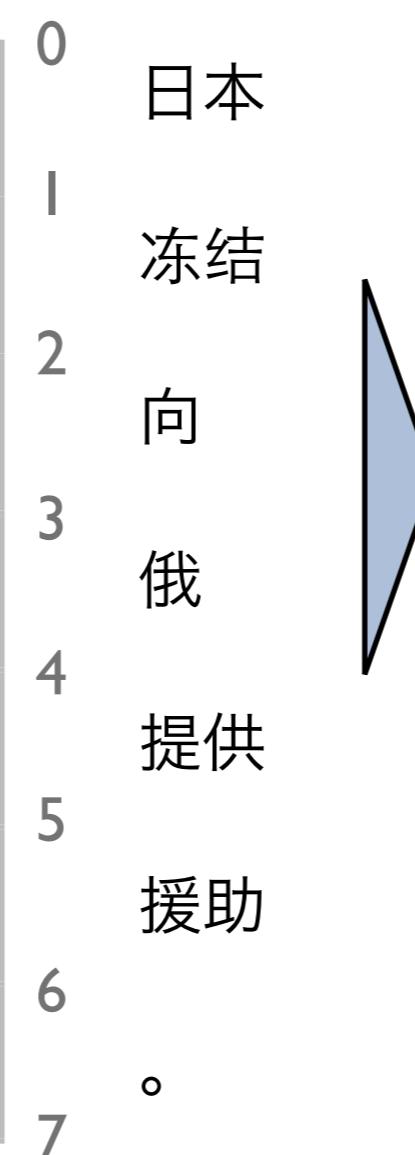
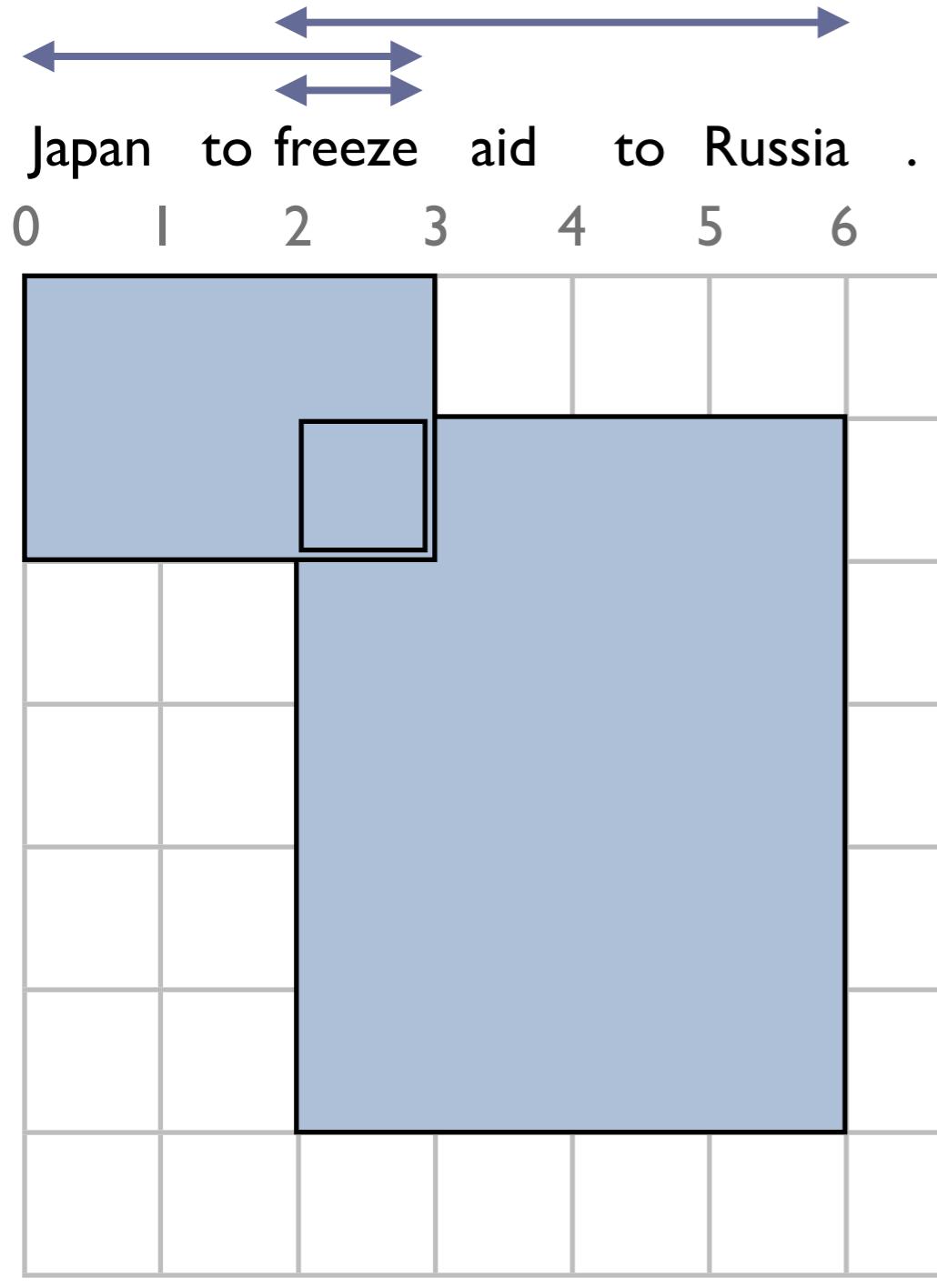
**e**: English segmentation

**f**: foreign segmentation

*Constants:*

**w**: weights

# Phrase Alignment as Integer Programming



$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$

$$\text{s.t. } \sum_{i,j:i < x \leq j} e_{i,j} = 1$$

$$\sum_{k,l:k < y \leq l} f_{k,l} = 1$$

*Indicator Variables:*

**a:** phrase alignment    **e:** English segmentation    **f:** foreign segmentation

*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming

Japan to freeze aid to Russia .

1 2 3 4 5 6


日本  
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。

$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$

$$\text{s.t. } \sum_{i,j: i < x \leq j} e_{i,j} = 1$$

$$\sum_{k,l: k < y \leq l} f_{k,l} = 1$$

*Indicator Variables:*

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*Constants:*

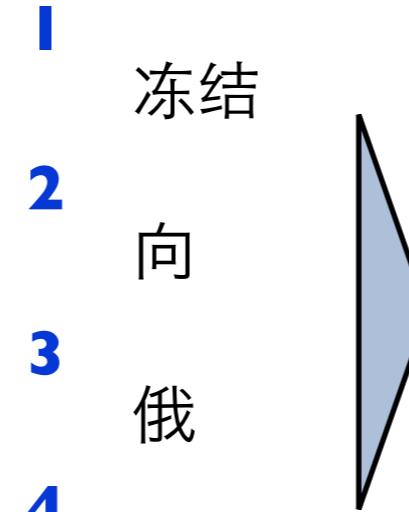
**w:** weights

# Phrase Alignment as Integer Programming

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*Constants:*

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*Indicator Variables:*

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# Phrase Alignment as Integer Programming

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$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$

$$\text{s.t. } \sum_{i,j: i < x \leq j} e_{i,j} = 1$$

$$\sum_{k,l: k < y \leq l} f_{k,l} = 1$$

$$\sum_{k,l} a_{i,j,k,l} = e_{i,j}$$

*Indicator Variables:*

**a:** phrase alignment    **e:** English segmentation    **f:** foreign segmentation

*Constants:*

**w:** weights

# Phrase Alignment as Integer Programming

# Japan to freeze aid to Russia .

1 2 3 4 5 6


日本

## 冻结

向

俄

# 提供

# 援助

○

$$\max \sum_{i,j,k,l} w_{i,j,k,l} \cdot a_{i,j,k,l}$$

$$\text{s.t.} \quad \sum_{i,j:i < x \leq j} e_{i,j} = 1$$

$$\sum_{k,l:k < y \leq l} f_{k,l} = 1$$

$$\sum_{k,l} a_{i,j,k,l} = e_{i,j}$$

$$\sum_{i,j} a_{i,j,k,l} = f_{k,l}$$

## *Indicator Variables:*

## a: phrase alignment

## e: English segmentation

**f:** foreign segmentation

## **Constants:**

## w: weights