Fully Distributed EM for Very Large Datasets

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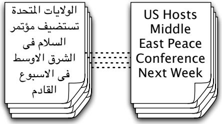
Overview

- Task: unsupervised learning via EM



- Focus: models w/ many local parameters
  (relevant to few datums)

- Approach: fully distributed, localized EM
  - ⋆ *parameter locality* → less bandwidth

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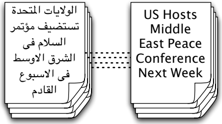

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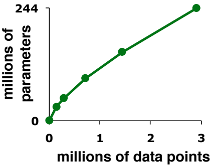

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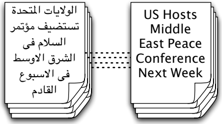

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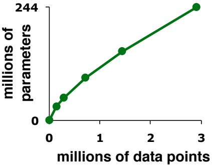

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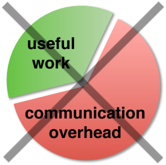

Outline

- Running example: IBM Model 1 for word alignment
- Naive distributed EM
- Efficiently distributed EM

Word alignment for machine translation

- Goal: parallel sentences $\longrightarrow$
  word-level translation model
- Parameters $\theta_{s \to t}$:
  probability that Spanish word s
  translates to English word t

$$\boldsymbol{\theta} = \left\{ \begin{array}{l} \theta_{\text{la}\to\text{the}} \\ \theta_{\text{la}\to\text{chair}} \\ \theta_{\text{la}\to\text{table}} \\ \hline \theta_{\text{silla}\to\text{the}} \\ \theta_{\text{silla}\to\text{chair}} \\ \hline \theta_{\text{mesa}\to\text{the}} \\ \theta_{\text{mesa}\to\text{table}} \end{array} \right.$$

la silla $\Downarrow$ **the chair**

la mesa $\Downarrow$ **the table**

corpus of parallel sentences

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corpus of parallel sentences

possible alignment arcs

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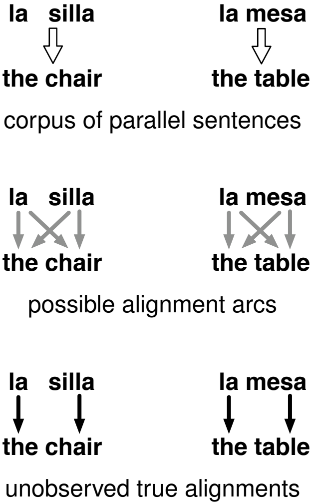

IBM Model 1 for word alignment

a Steve no le gustan las ferias grandes

? ? ? ? ? ? ?

each target word is generated by exactly
one source word chosen u.a.r

- IBM Model 1: a simple generative model
  - For each target position i, independently
    - choose a source index a_i u.a.r.
    - choose a target word $T_i \sim \theta_{\mathrm{S}_{\mathrm{a_i}}\rightarrow\cdot}$

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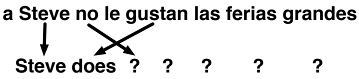

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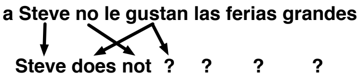

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EM algorithm for IBM Model 1

- $\boldsymbol{\theta} \leftarrow$ some initial guess

$\theta_{\text{la}\to\text{the}}$=.33, $\theta_{\text{la}\to\text{chair}}$=.33,
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- Iterate:
  1. E-step: estimate alignment counts $\boldsymbol{\eta}$
     1. compute posteriors $p(a_i|\boldsymbol{\theta})$

la **silla**

$\frac{.33}{.33+.5} = .4$ $\quad$ $.6 = \frac{.5}{.33+.5}$

the chair

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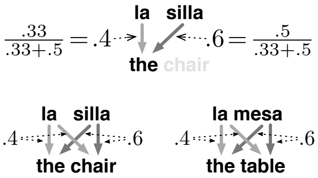

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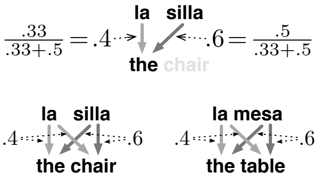
     2. aggregate into expected counts $\eta_{s\to t}$ (expected # times $s \to t$ under $\boldsymbol{\theta}$)

$$\eta_{s\to t} \leftarrow \sum_{\mathcal{C}} \frac{\theta_{s\to t}}{\sum_{i'} \theta_{s_{i'}\to t}}$$



$\eta_{\text{la}\to\text{the}}$=.8, $\eta_{\text{la}\to\text{chair}}$=.4,
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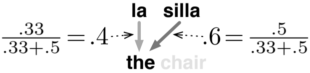

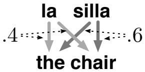

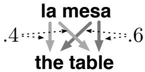

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  2. M-step: normalize $\boldsymbol{\eta}$ to get new ML $\boldsymbol{\theta}$

$$\theta_{s\to t} \leftarrow \frac{\eta_{s\to t}}{\sum_{t'} \eta_{s\to t'}}$$

$\theta_{\text{la}\to\text{the}}$=.5, $\theta_{\text{la}\to\text{chair}}$=.25,
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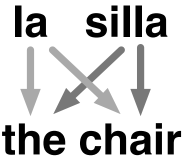
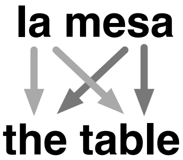
EM example continued

E-Step 1





E-Step 2

EM example continued

E-Step 3

E-Step 4

EM example continued

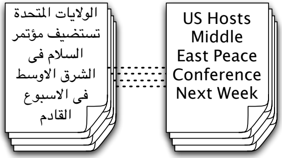
E-Step 5

EM example continued

E-Step ∞

la	**silla**	**la**	**mesa**
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
the	**chair**	**the**	**table**

UN Arabic English TIDES v2 corpus



- 2.9 million sentence pairs from UN proceedings
- 243 million *unique* word pairs
  (translations possible in some sentence pair)
  - 243 M parameters in θ
  - 243 M counts in η
- Even fitting all (indexed) parameters in
  32-bit memory can be challenging

Outline

- Running example: IBM Model 1 for word alignment
- Naive distributed EM
- Efficiently distributed EM

Previous approach: distributing the E-step

1. E-step computations distribute easily
   - partition data over k nodes
   - alignments independent given θ
2. Nodes communicate partial counts to central Reduce node
3. Reduce node does global M-step
4. Reduce sends new parameters back

- Remaining problems:
  - Memory at Reduce node
  - C-step (communication) bandwidth: 5.5 B numbers per iteration (on full dataset with 20 nodes)

(Chu *et al.* 2006, Dyer *et al.* 2008, Newman *et al.* 2008, ...)

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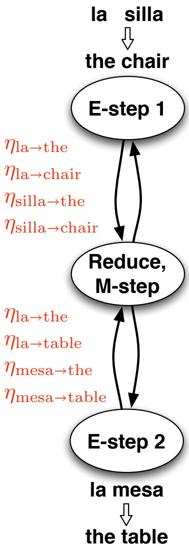

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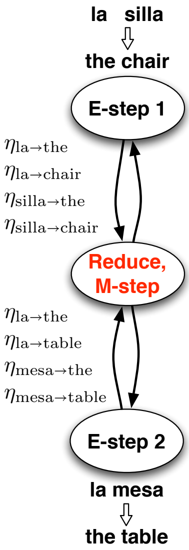

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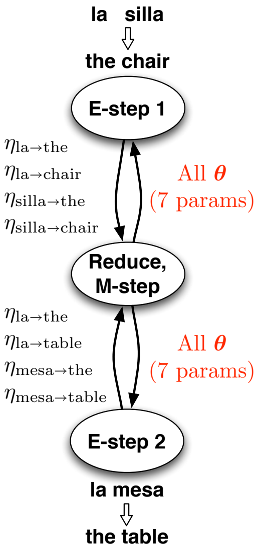

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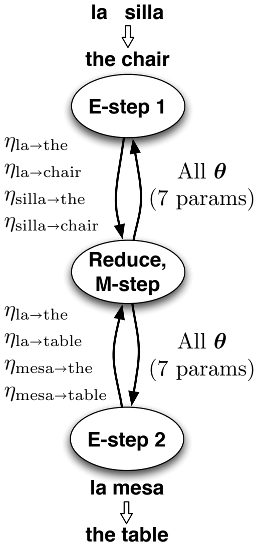

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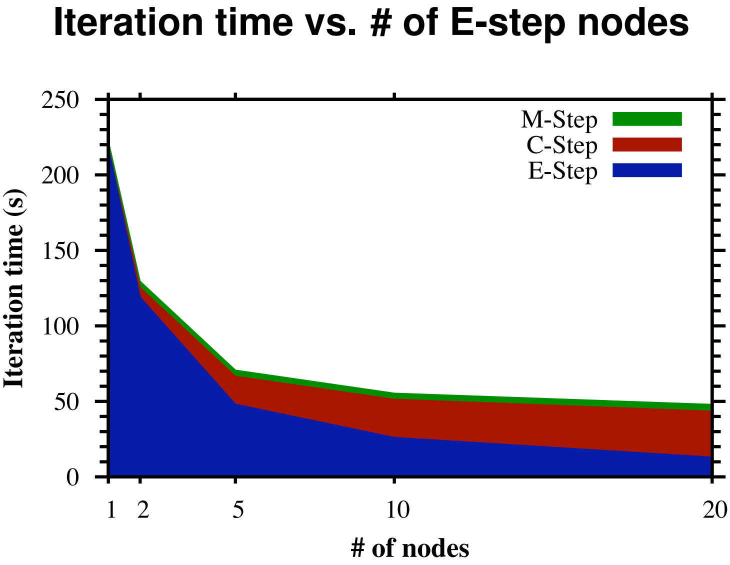
Speedup (on 200K total sentence pairs)



Common practical solutions

- Memory and bandwidth are real problems in practice
- Workarounds
  - Use less data
  - Ignore rare words
  - Train on independent chunks
  - Swap to disk
  - Distribute over multiple machines

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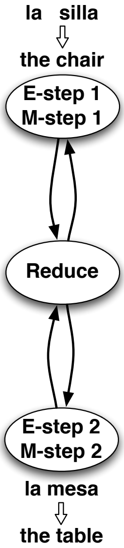
Distributing the M-step locally



- Distribute M-step alongside E-step
  - Nodes store only needed params, compute them locally
  - Reduce passes back counts
  - Don't need to hear about irrelevant source words
  - Don't need to tell (or hear) about purely local source words
  - Need to hear everything about each source word: M-step denominator

$$\theta_{s \to t} \leftarrow \frac{\eta_{s \to t}}{\sum_{t'} \eta_{s \to t'}}$$

  - Bandwidth savings: 30%

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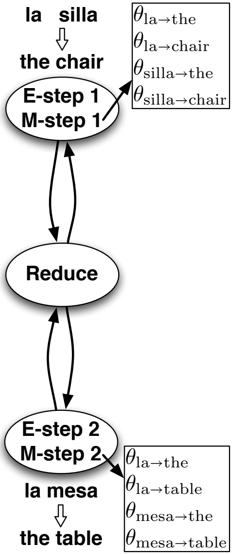

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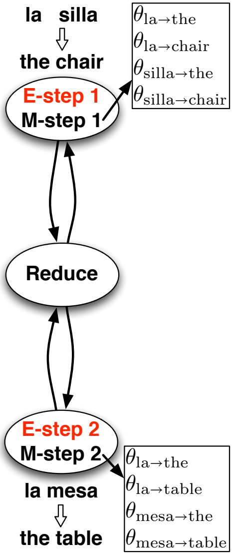

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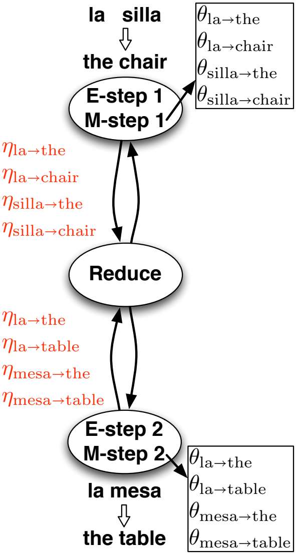

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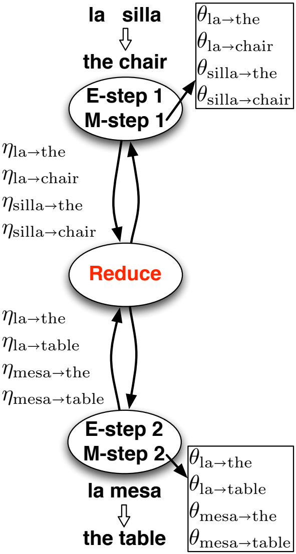

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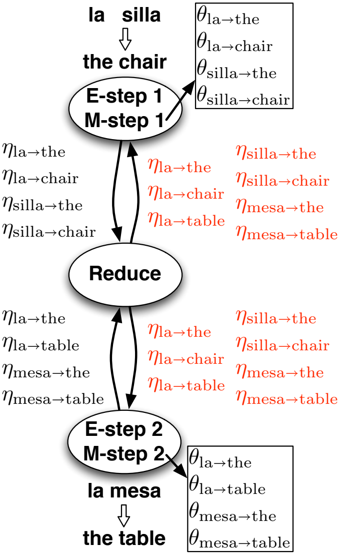

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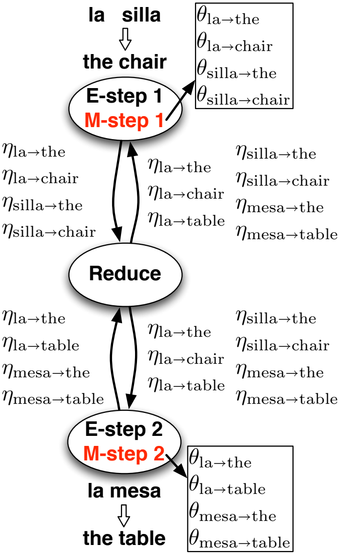

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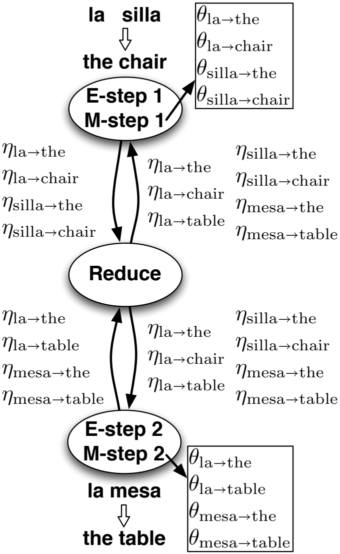
  - Need to hear everything about each source word: M-step denominator

$$\theta_{s\to t} \leftarrow \frac{\eta_{s\to t}}{\sum_{t'} \eta_{s\to t'}}$$

  - Bandwidth savings: 30%

Distributing the M-step locally



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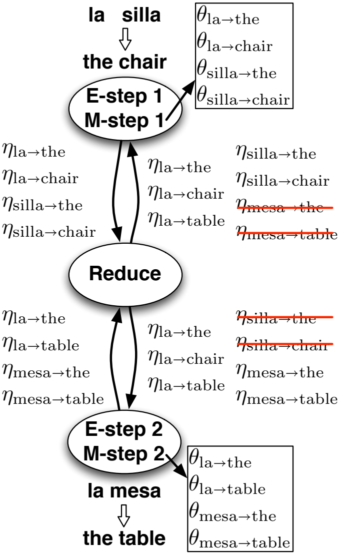

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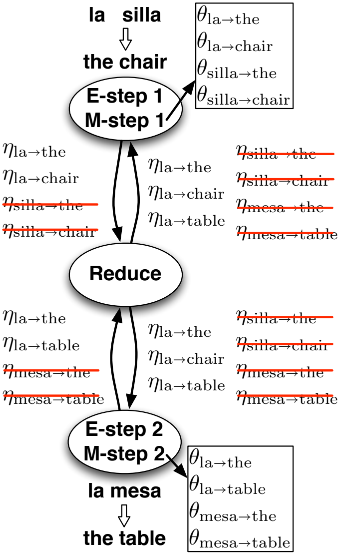

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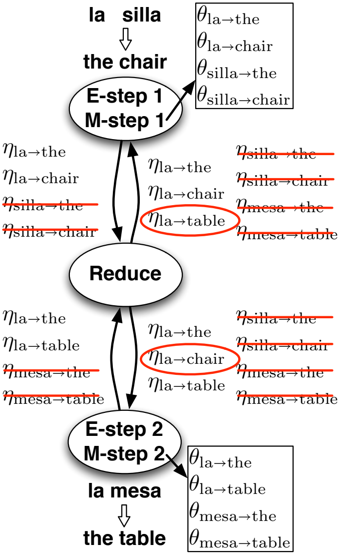

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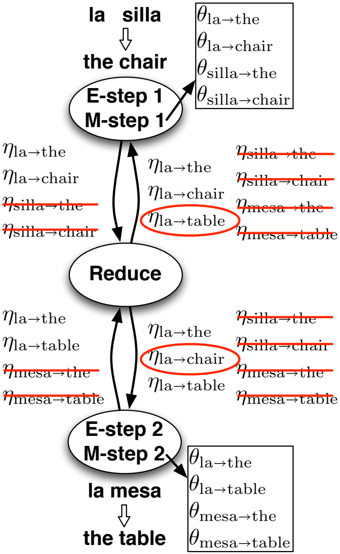

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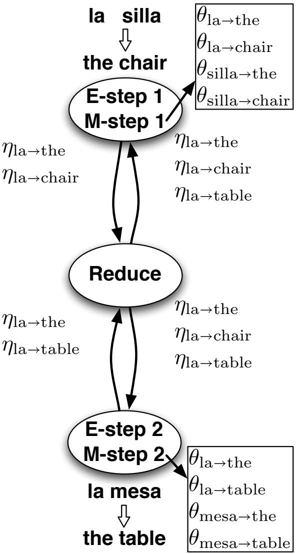
Augmenting η to increase locality



- Augment η with redundant $\eta_s = \sum_{t'} \eta_{s\to t'}$ in E-step
- M-step becomes $\theta_{s\to t} \leftarrow \frac{\eta_{s\to t}}{\eta_s}$
- Increases locality
- Total bandwidth savings: 84% (bigger if more nodes)
- Similar tricks for other models

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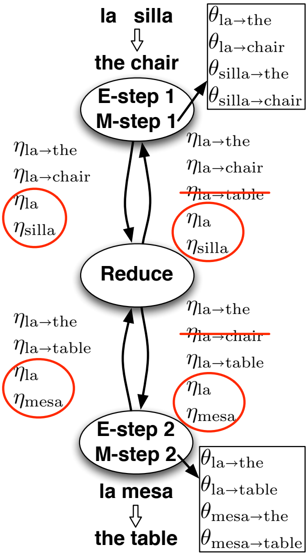

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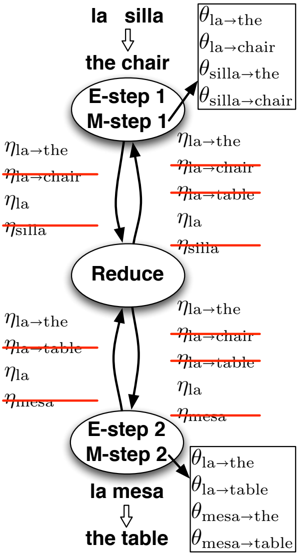

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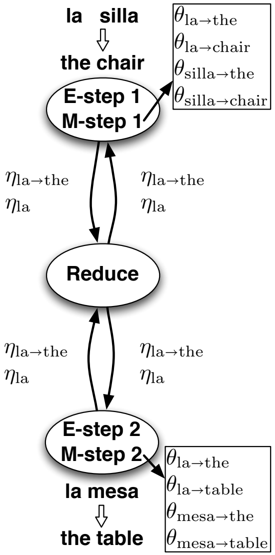

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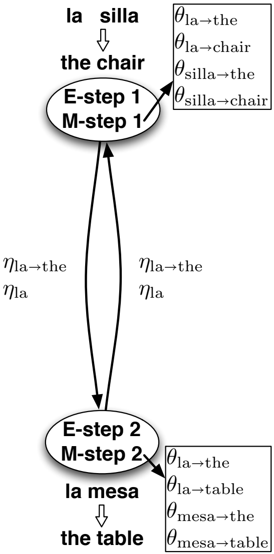
Choice of C-step topology



- No need for separate Reduce nodes
- By choosing connectivity, can trade off
  - bandwidth
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  - ...

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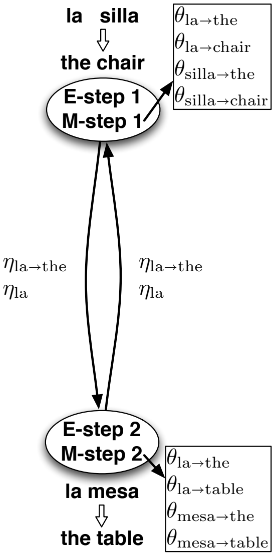

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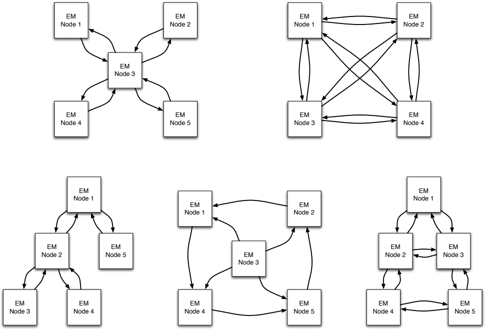
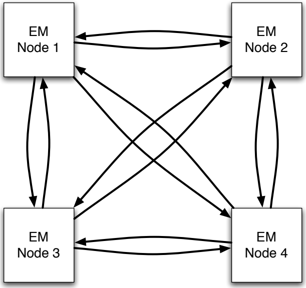
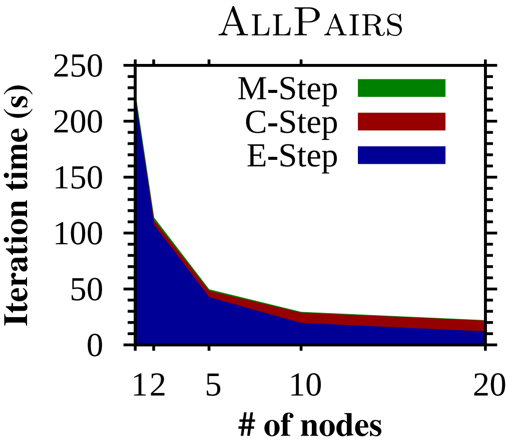


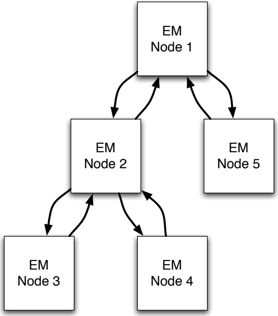
AllPairs topology





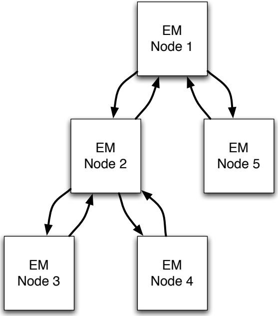
Total Bandwidth: 3.6 B counts per iteration

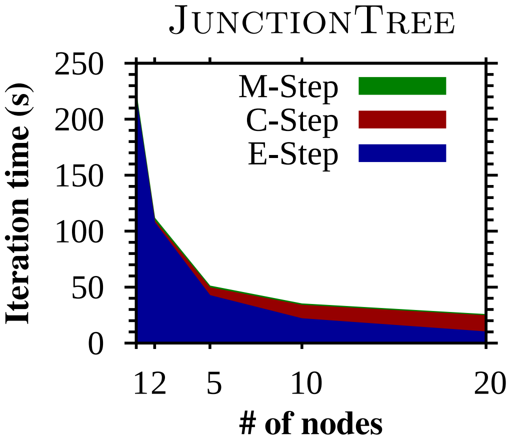
JunctionTree topology



- Nodes embedded in arbitrary tree structure
- Messages contain counts needed by nodes in both subtrees
- Tree can optimize for
  - bandwidth
  - locality
  - ...
- We use maximum spanning tree to heuristically minimize bandwidth
- Future work: multiple trees

JUNCTIONTREE topology





Total Bandwidth: 1.4 B counts per iteration

Locality in other models

- Ex: Latent Dirichlet Allocation (LDA) for topic modeling
  - Parameters: unigram distributions for each topic $p(w|t)$
  - Topic-word parameters local
  - Similar augmentation trick to Model 1
  - Details and results in paper
- Also applies to other EM models, beyond EM
  - Word locality is extremely common in NLP applications
  - Variational inference
  - Other computations that make sparse use of expectations

Conclusion

- A fully distributed, maximally localized EM algorithm
  - exploits parameter locality for significant speedup
  - is general; just define η for each datum
  - is flexible with respect to communication topology
- Many further improvements possible
  - intelligent partitioning of data
  - running E- and C-steps in parallel
  - better topologies (e.g., multiple trees)
  - exploiting approximate sparsity/locality
  - ...