

# Online EM for Unsupervised Models

NAACL – June 3, 2009

Percy Liang

Dan Klein



# Based on a true story

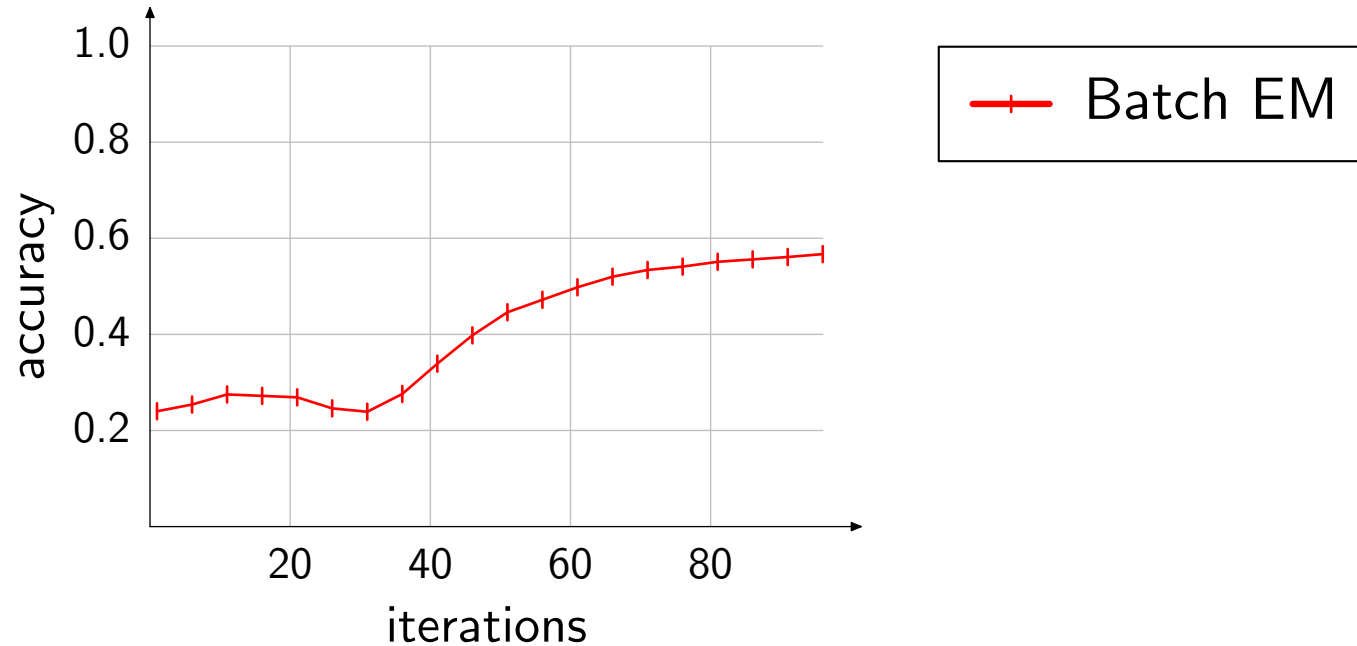
Part-of-speech induction:

DT    NNP        NNP    VBD  
*The European Commission agreed*

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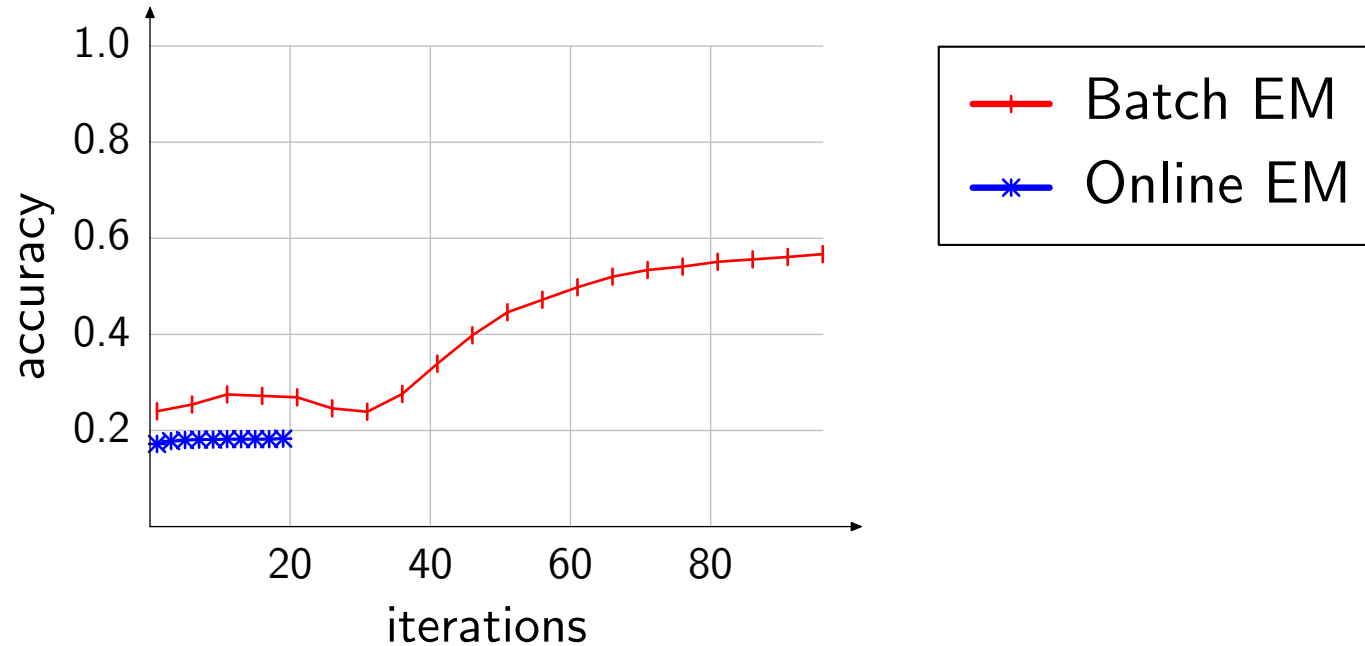
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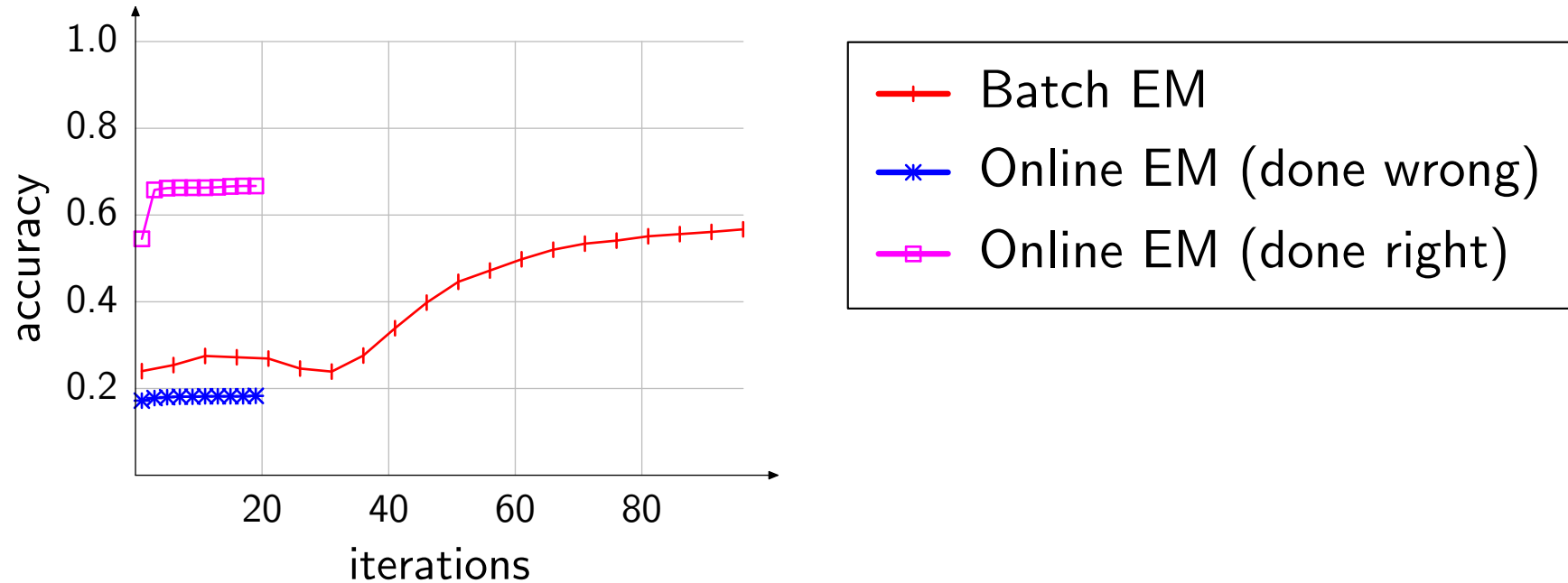
DT    NNP            NNP    VBD  
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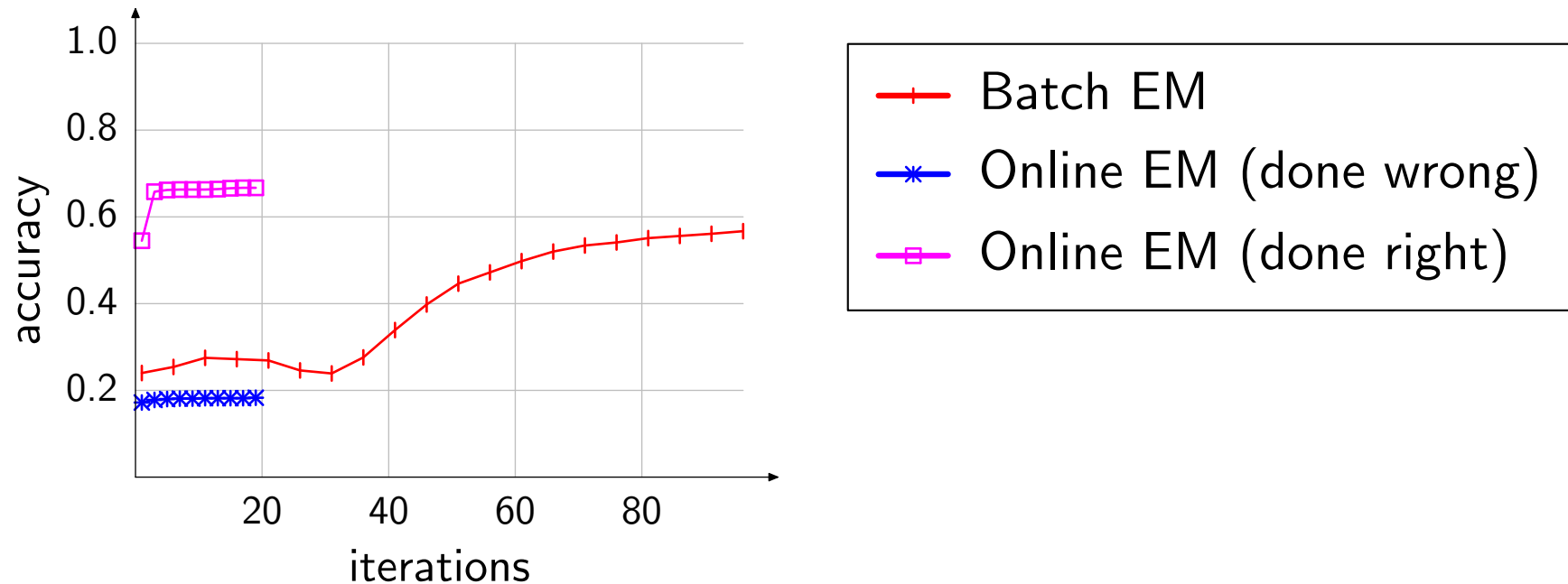
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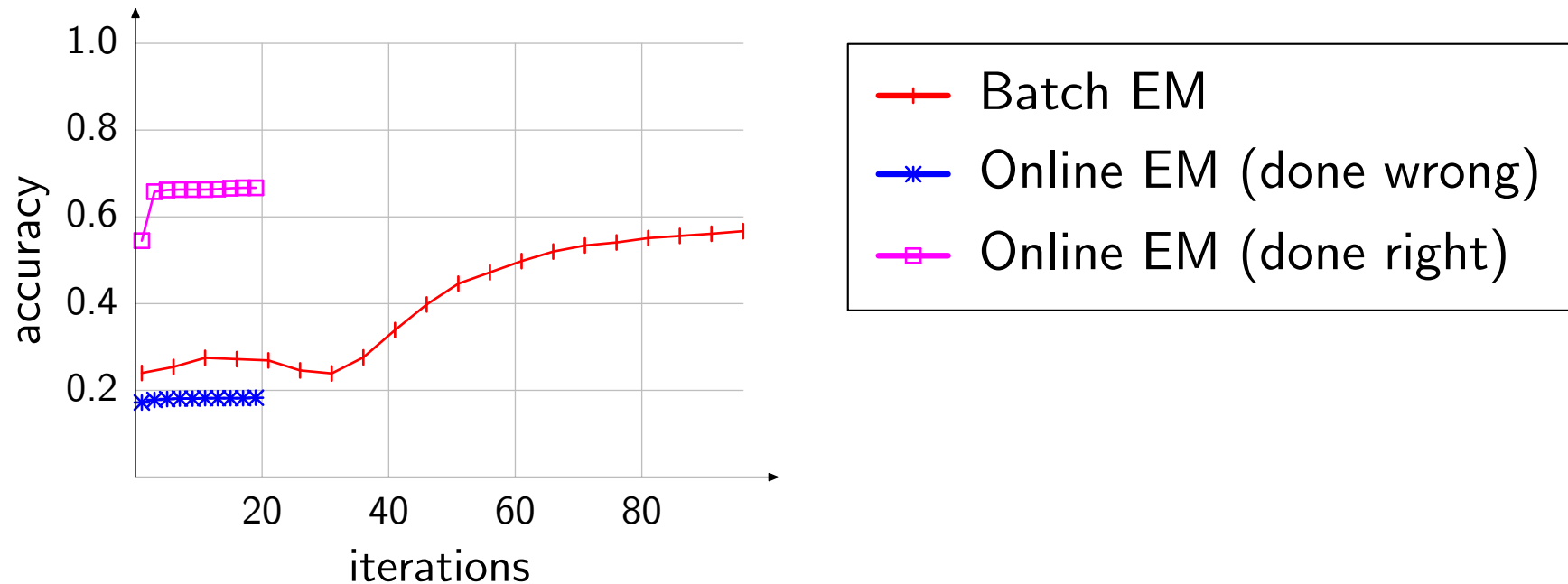
Observations:

1. Online EM is faster than batch EM

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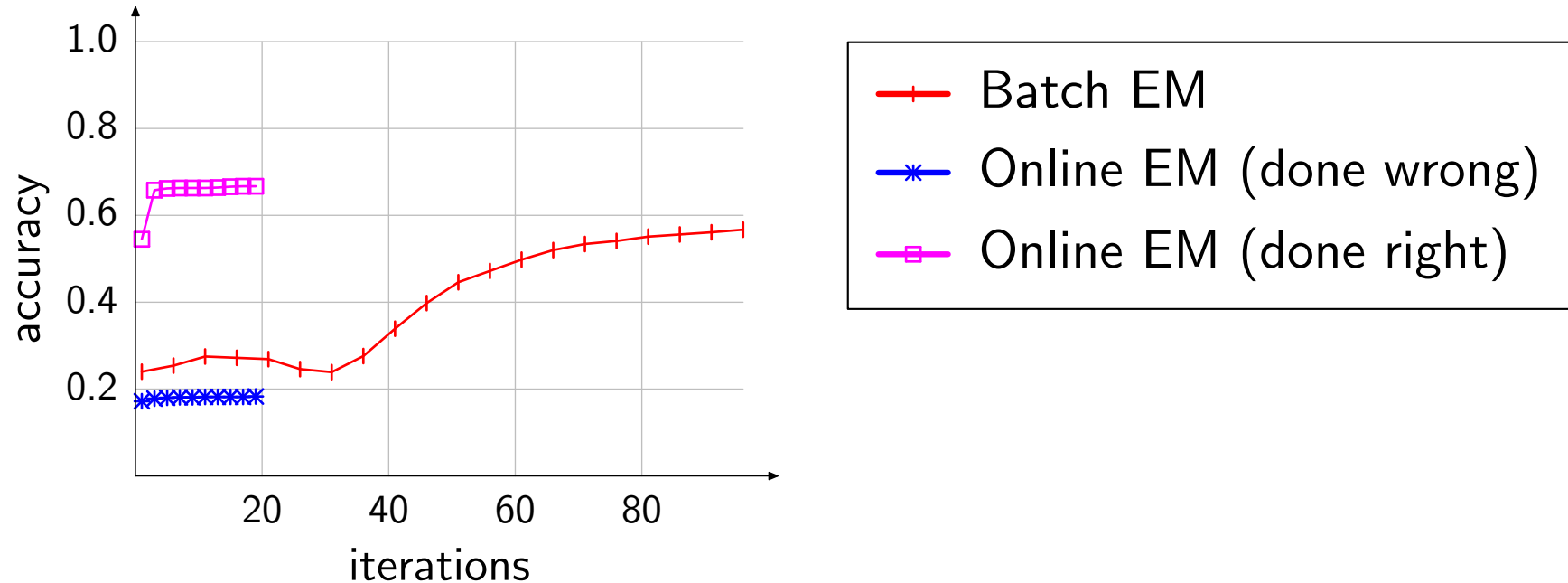
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# Based on a true story

Part-of-speech induction:

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Observations:

1. Online EM is faster than batch EM
2. Online EM improves accuracy(!)
3. Details of online EM do matter



# Four tasks

DT   NNP   NNP   VBD  
*The European Commission agreed*

POS tagging

# Four tasks

DT    NNP        NNP        VBD  
*The European Commission agreed*

POS tagging

*l o o k | a t | t h e | b o o k*

Word segmentation

# Four tasks

DT    NNP            NNP    VBD  
*The European Commission agreed*

POS tagging

*l o o k | a t | t h e | b o o k*

Word segmentation

## BASEBALL

*...Matt Williams has demonstrated throughout  
his career that he will NOT wait for good  
pitches to hit...*

Document classification

# Four tasks

DT    NNP            NNP            VBD  
*The European Commission agreed*

POS tagging

*l o o k | a t | t h e | b o o k*

Word segmentation

## BASEBALL

*...Matt Williams has demonstrated throughout his career that he will NOT wait for good pitches to hit...*

Document classification

*the    European    Commission*  
*la    Commission    européenne*

Word alignment

# Unsupervised induction

Setting:

$$\mathbf{x}^{(1)} \quad \mathbf{x}^{(2)} \quad \dots \quad \mathbf{x}^{(n)}$$

# Unsupervised induction

Setting:

$$\begin{array}{cccc} \mathbf{x}^{(1)} & \mathbf{x}^{(2)} & \dots & \mathbf{x}^{(n)} \\ \mathbf{z}^{(1)} & \mathbf{z}^{(2)} & \dots & \mathbf{z}^{(n)} \end{array}$$

# Unsupervised induction

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Probabilistic model:  $p(\mathbf{x}, \mathbf{z}; \theta)$

$\mathbf{x}$ : observed input

$\mathbf{z}$ : hidden output

$\theta$ : parameters (multinomial probabilities)

# Unsupervised induction

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Training objective: **likelihood**

$$\theta^* = \operatorname{argmax}_{\theta} \sum_{i=1}^n \log p(\mathbf{x}^{(i)}; \theta)$$



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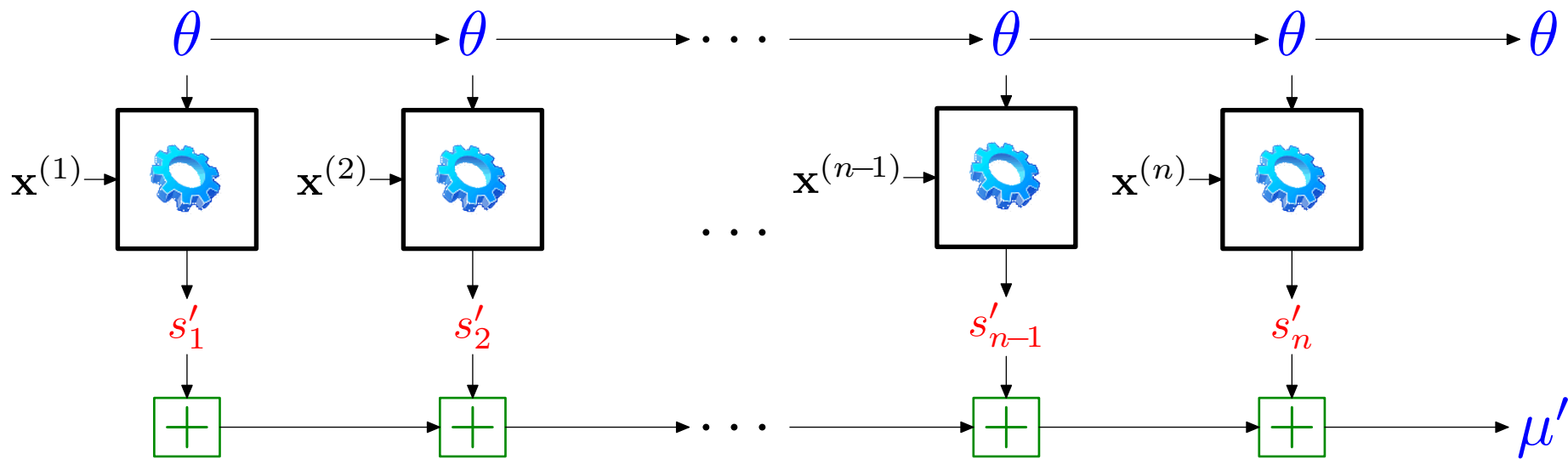
Training objective: **likelihood**

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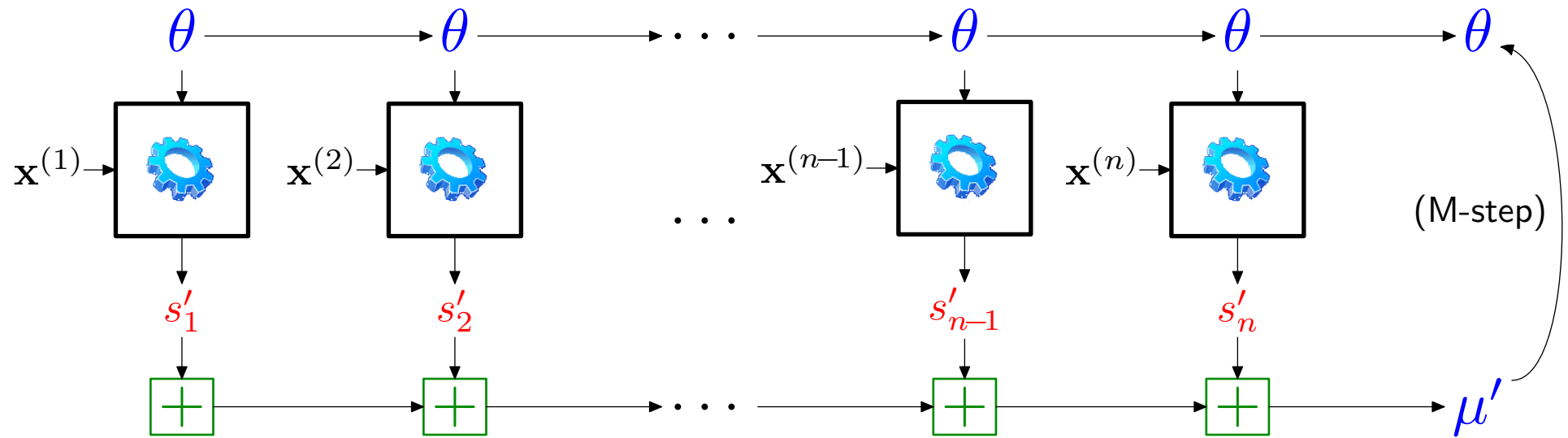
Evaluation: **accuracy**

gold  $\mathbf{z}^{(i)}$  versus predicted  $\operatorname{argmax}_{\mathbf{z}} p(\mathbf{z} | \mathbf{x}^{(i)}; \theta^*)$

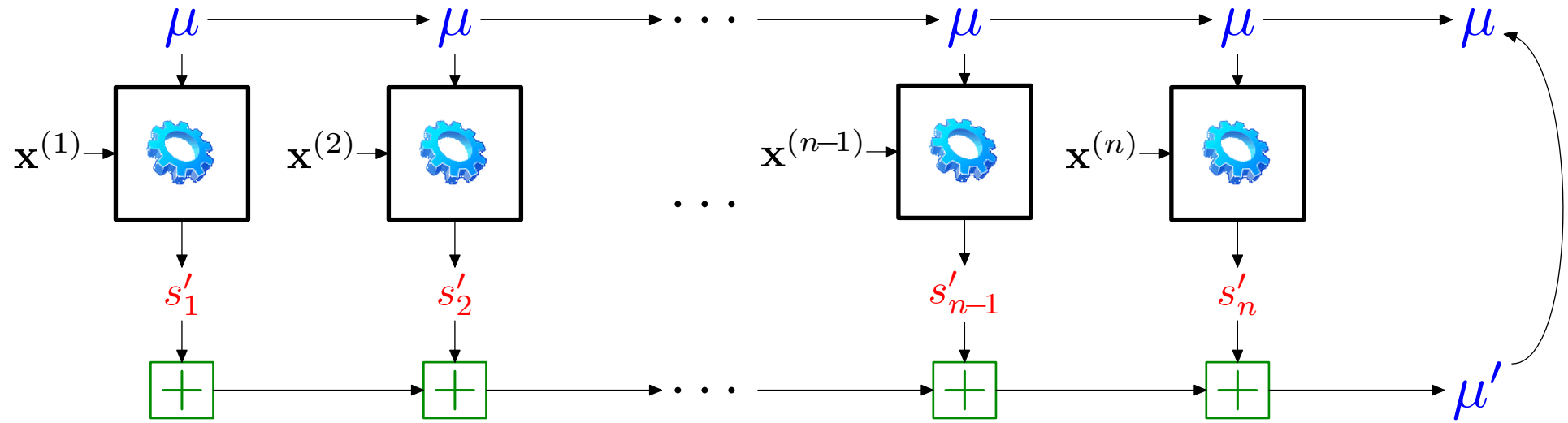
# Batch EM



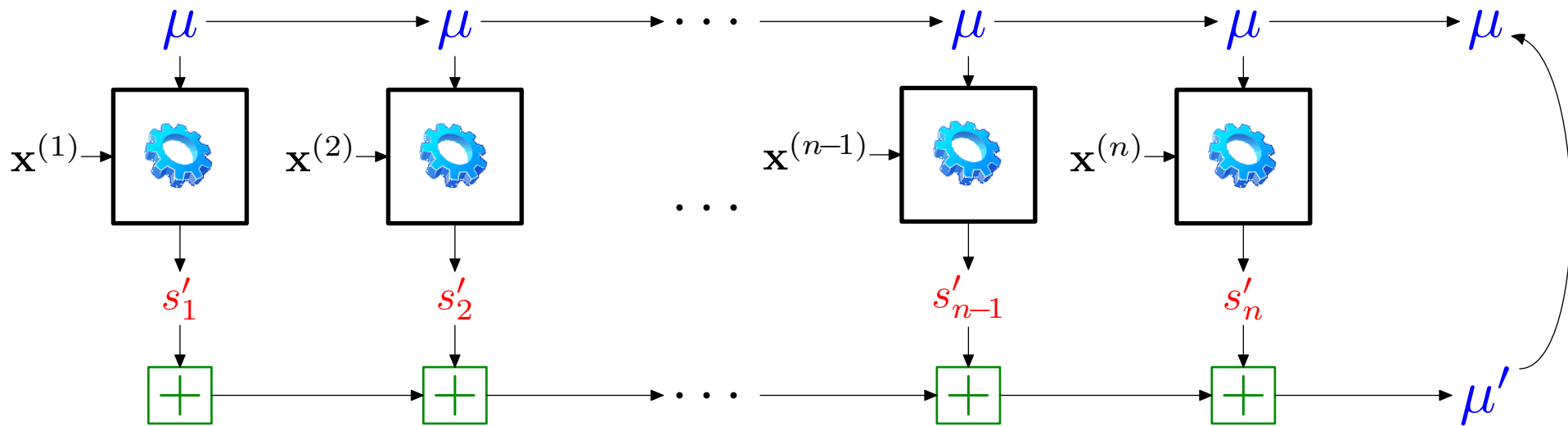
# Batch EM



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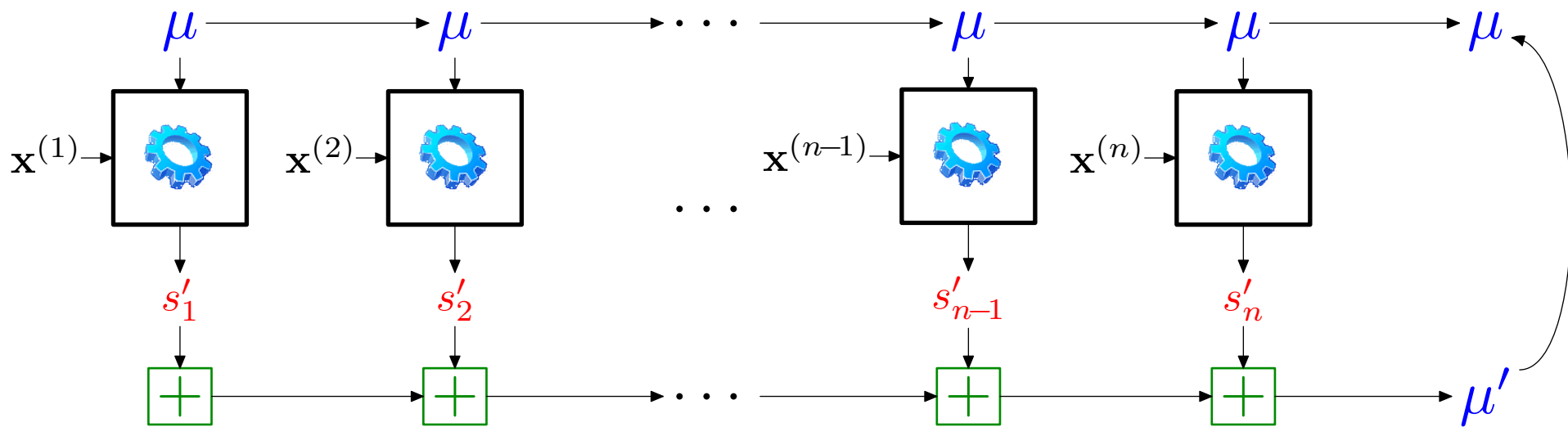
Data  
(0,8)  
(6,2)  
(3,8)  
(2,1)  
(3,5)  
(2,4)  
(4,4)  
(5,7)  
(3,6)  
(4,3)

parameter space

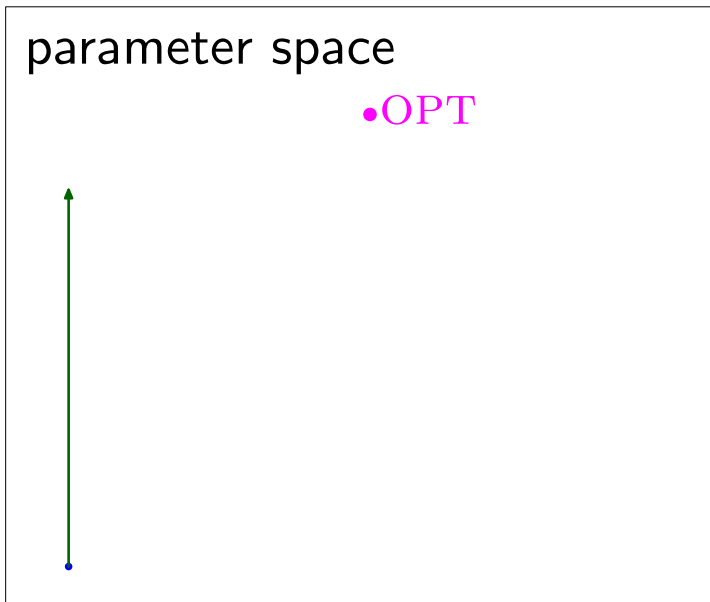
•OPT

0 data points processed

# Batch EM

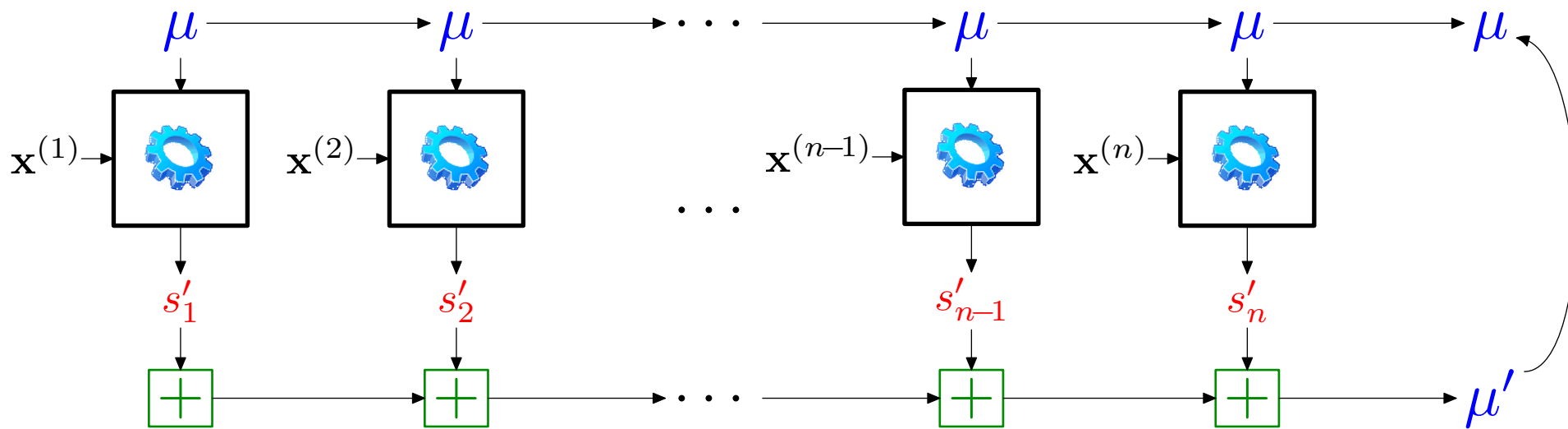


- Data
- (0,8)
  - (6,2)
  - (3,8)
  - (2,1)
  - (3,5)
  - (2,4)
  - (4,4)
  - (5,7)
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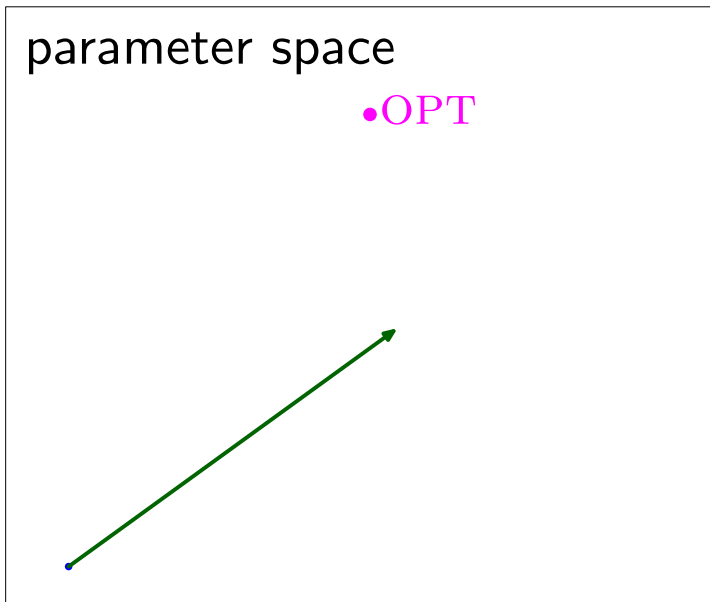


1 data points processed

# Batch EM

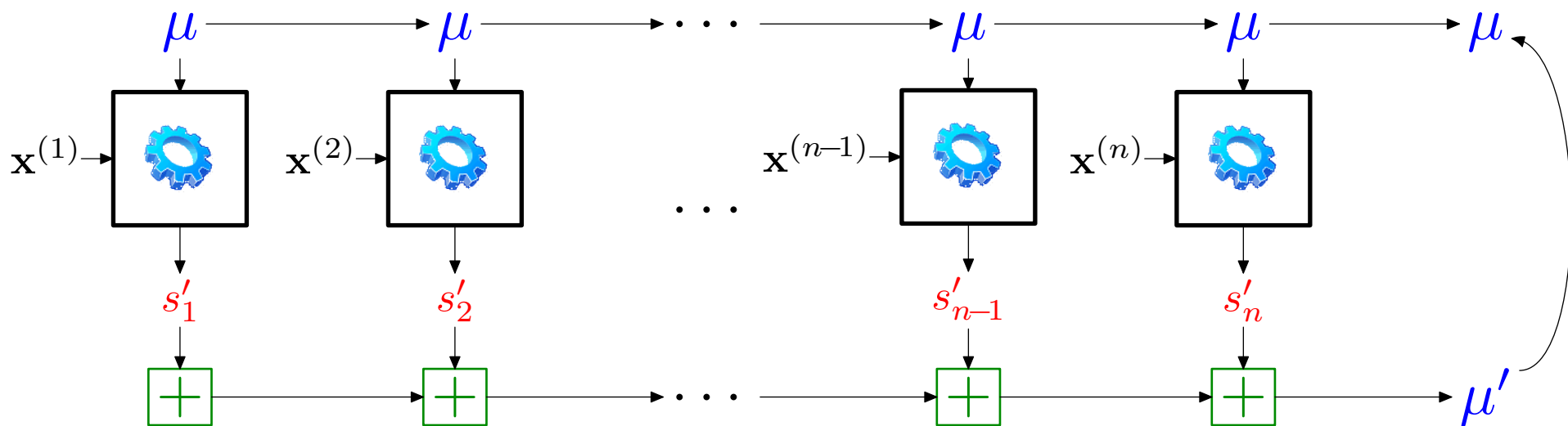


- Data
- (0,8)
- (6,2)
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- (3,5)
- (2,4)
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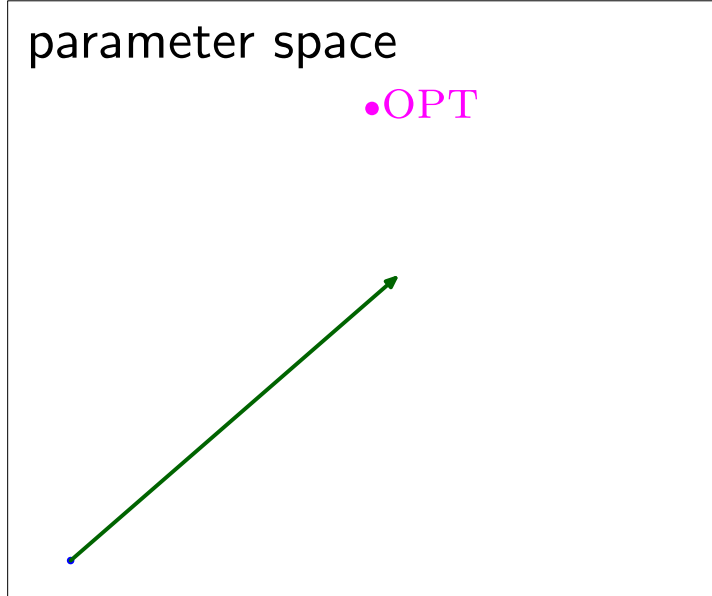
2 data points processed

# Batch EM



Data

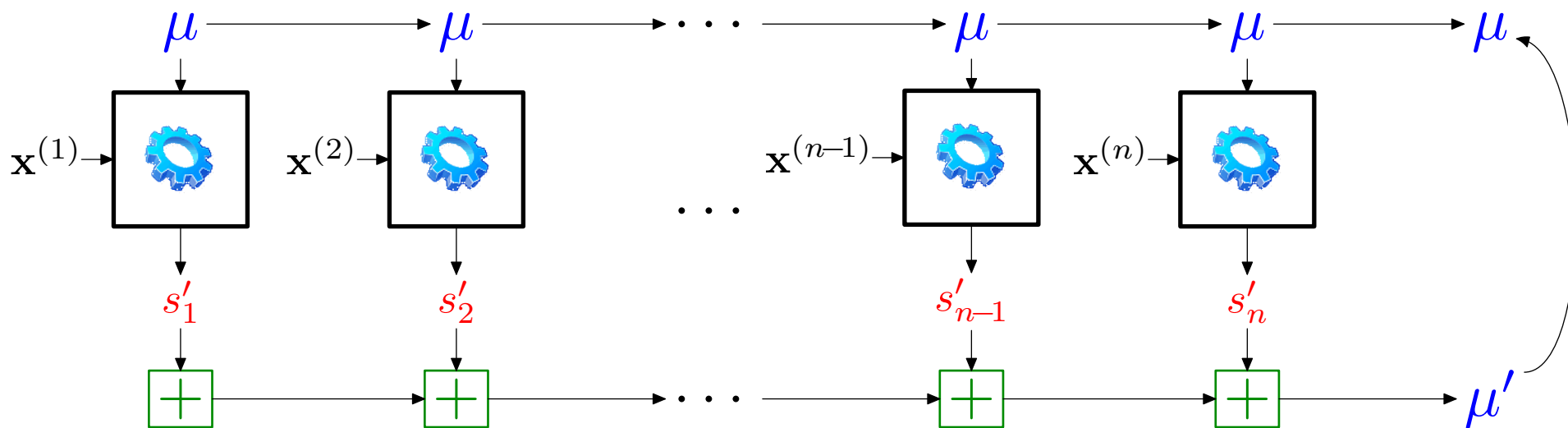
- (0,8)
- (6,2)
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- (2,1)
- (3,5)
- (2,4)
- (4,4)
- (5,7)
- (3,6)
- (4,3)



3 data points processed

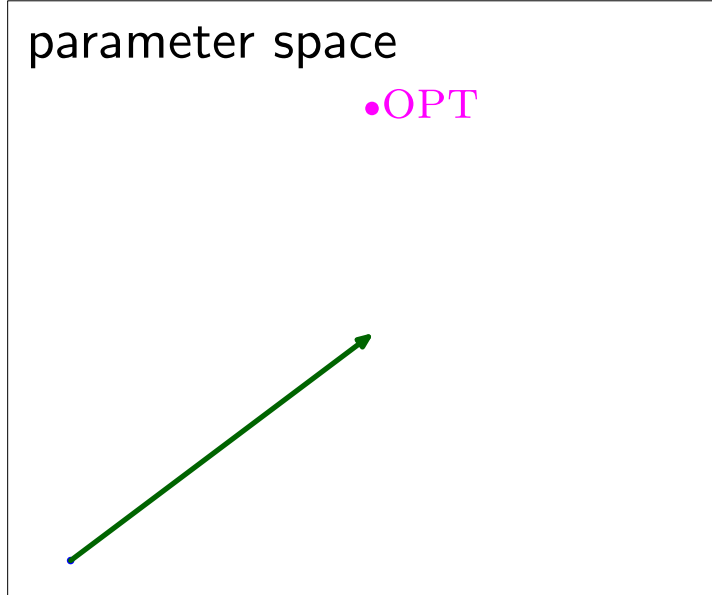


# Batch EM

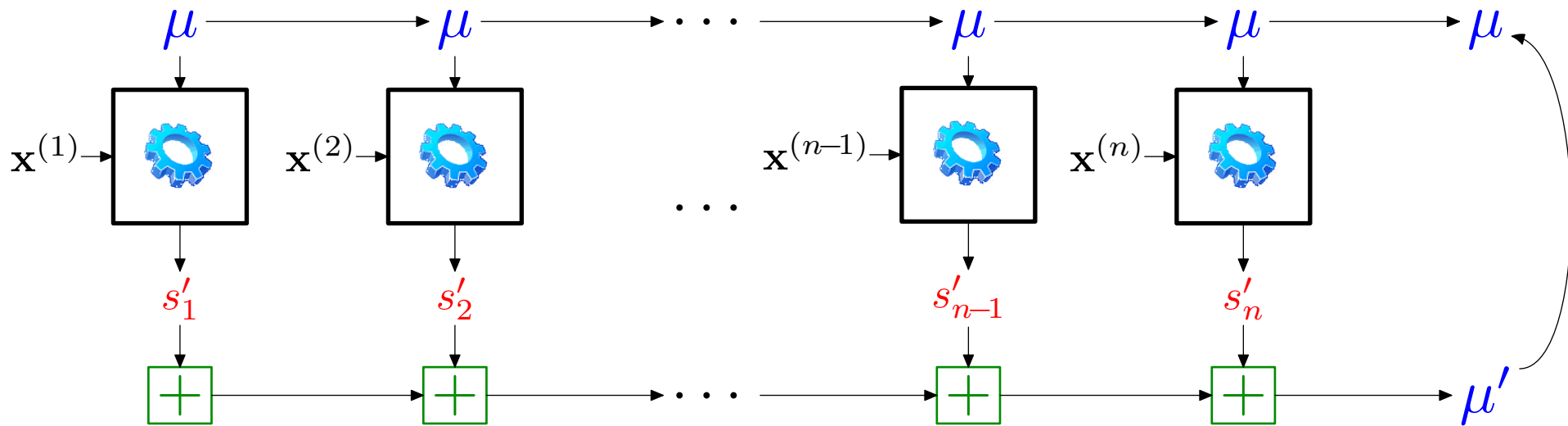


Data

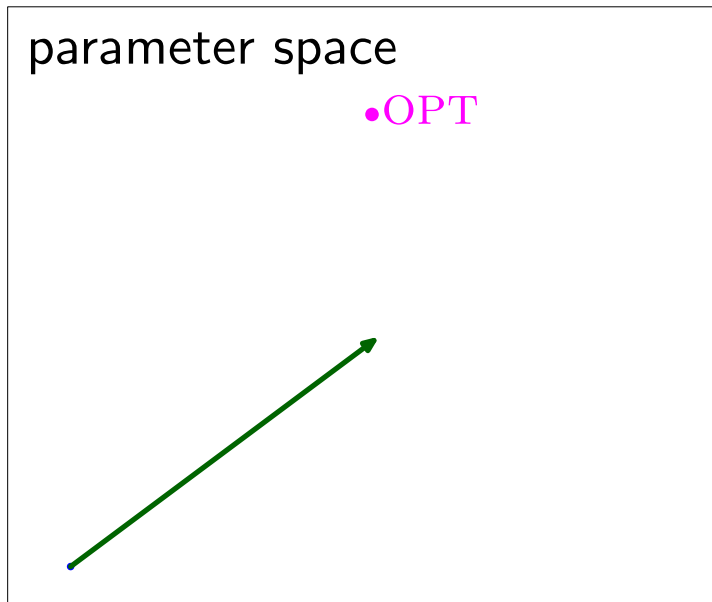
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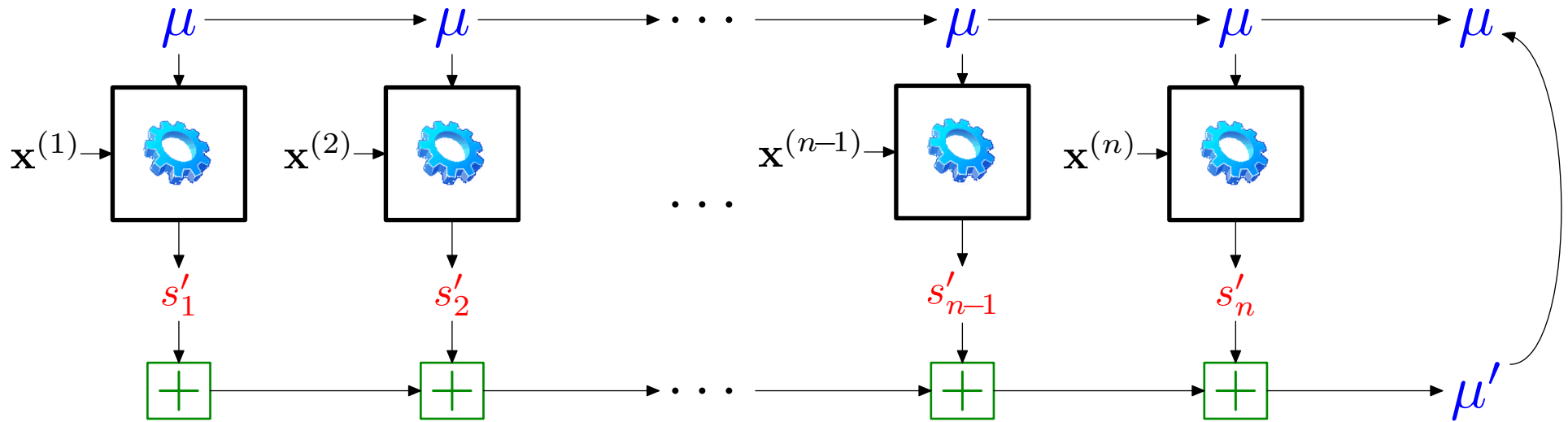
# Batch EM



Data  
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(2,1)  
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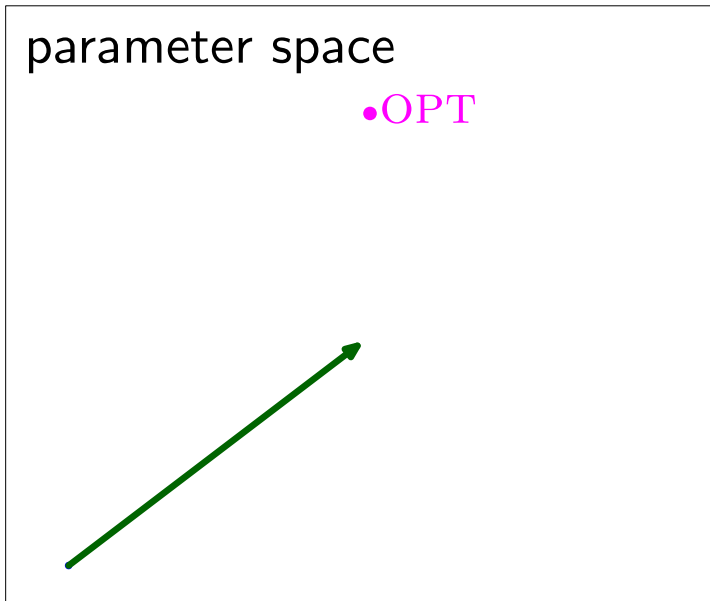


# Batch EM



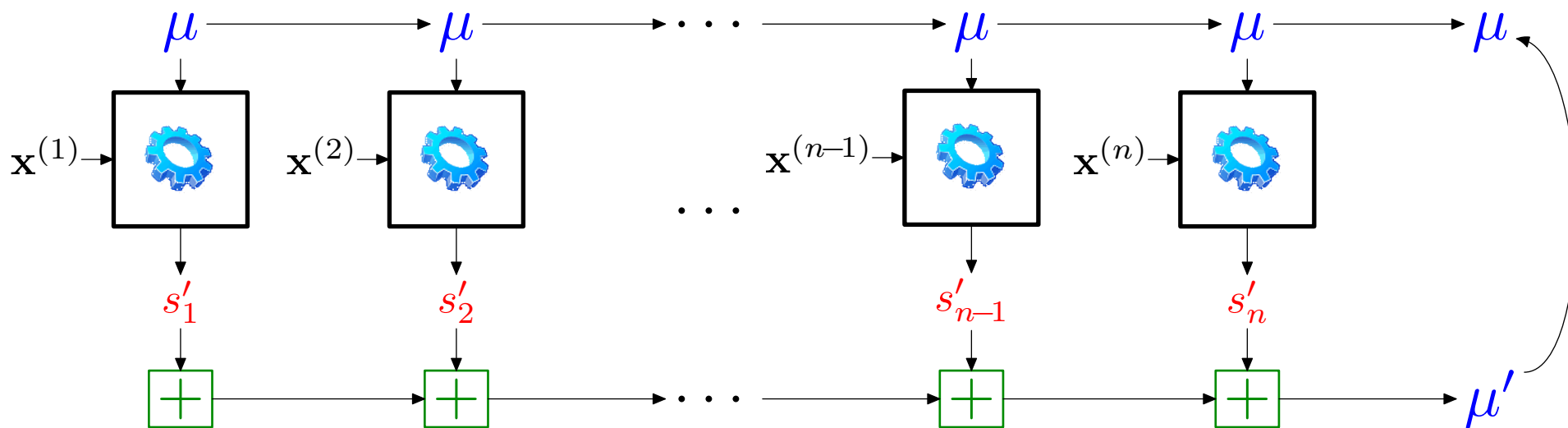
Data

- (0,8)
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- (2,4)
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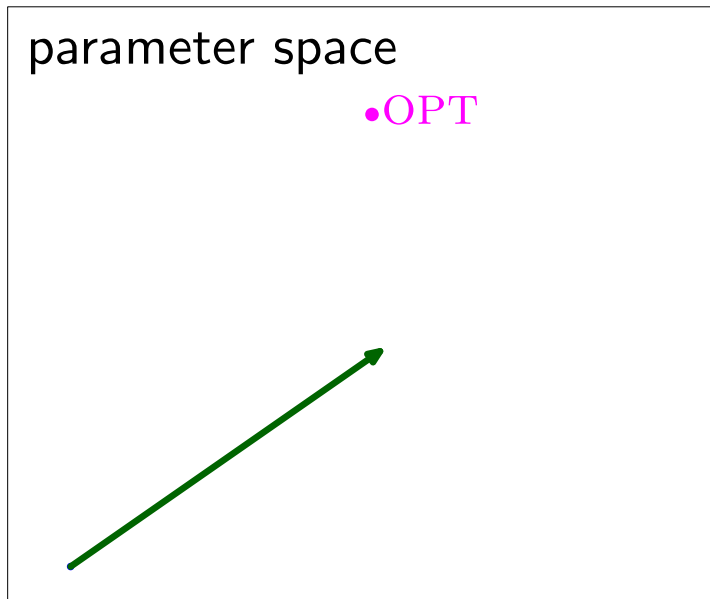


6 data points processed

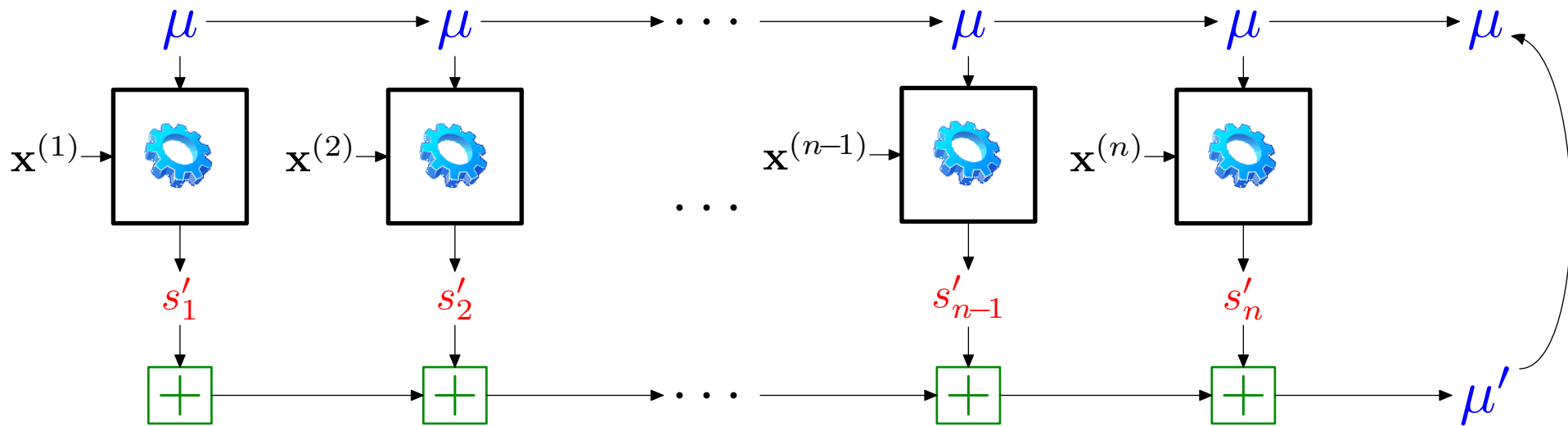
# Batch EM



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(0,8)  
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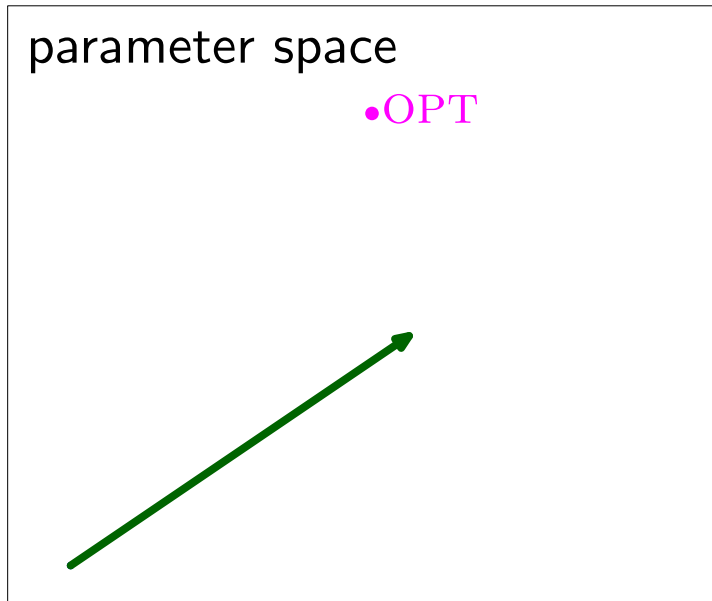


# Batch EM

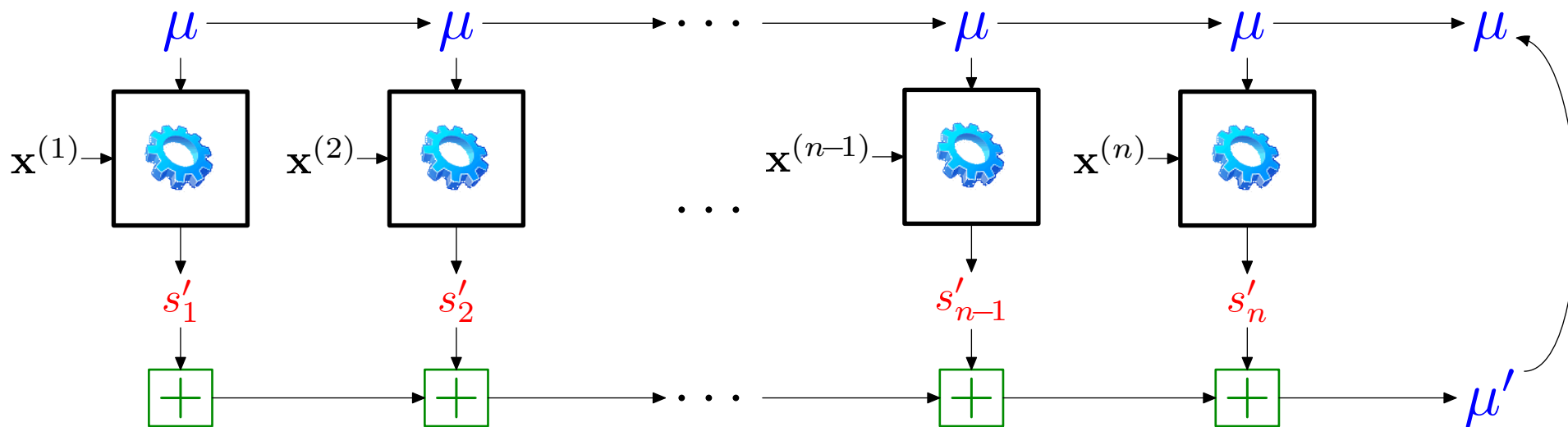


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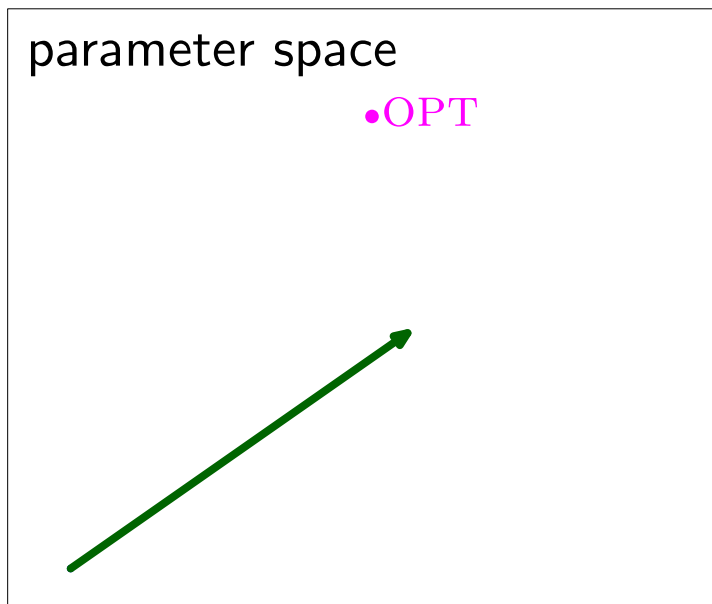


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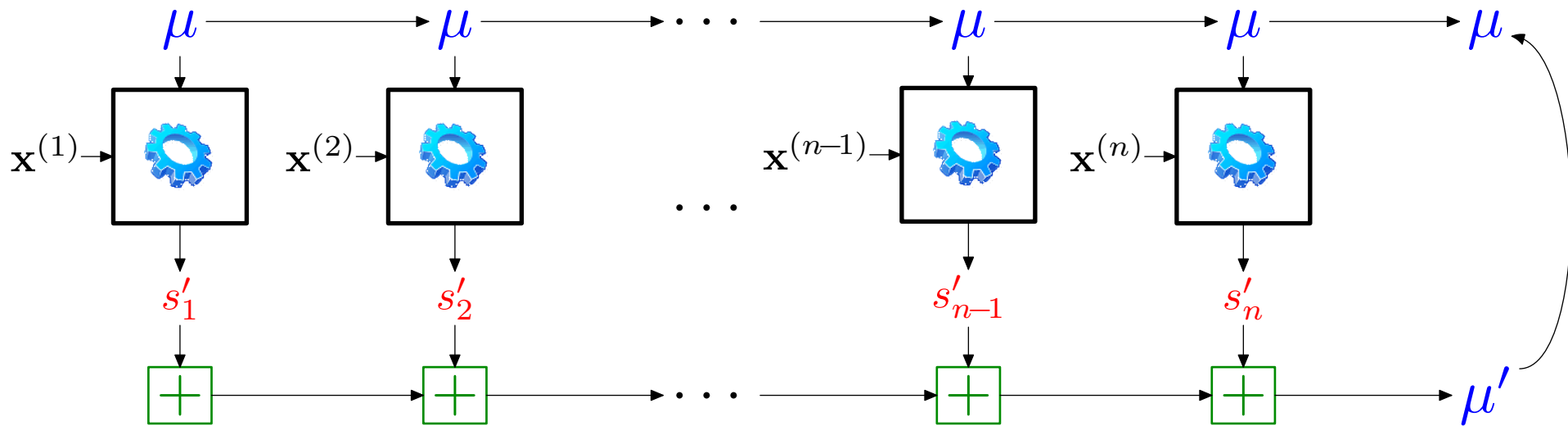


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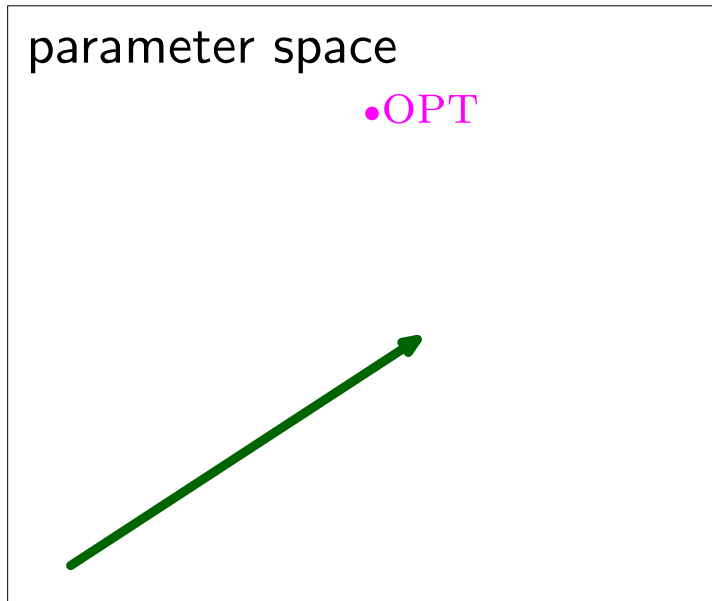


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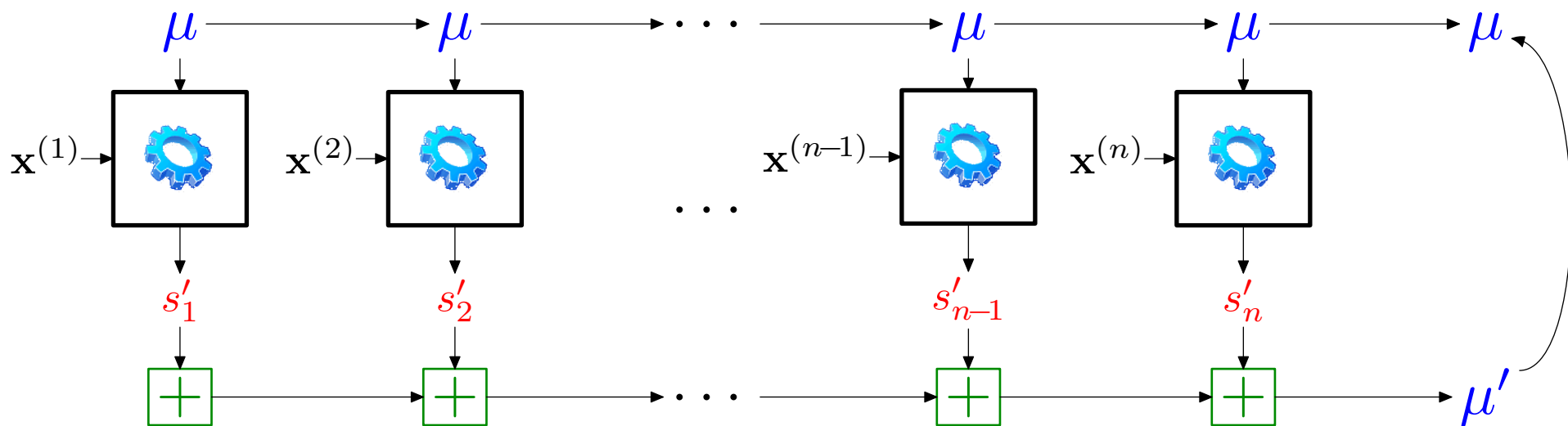
Data

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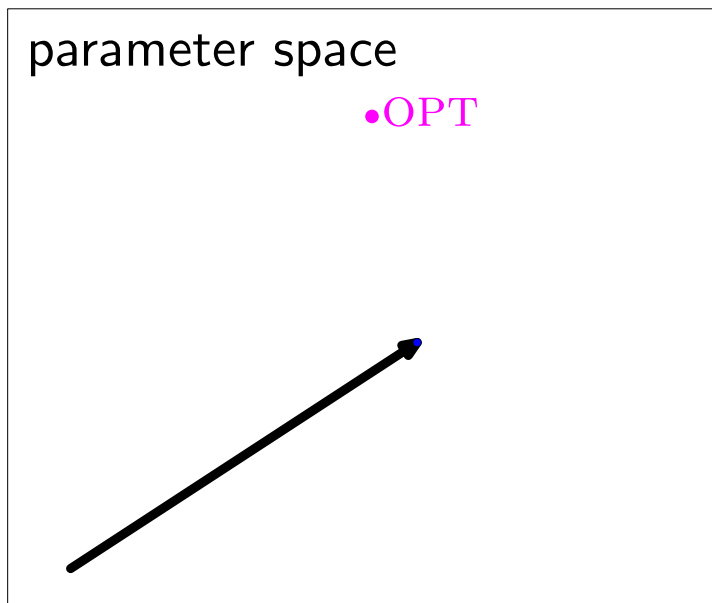
10 data points processed

# Batch EM



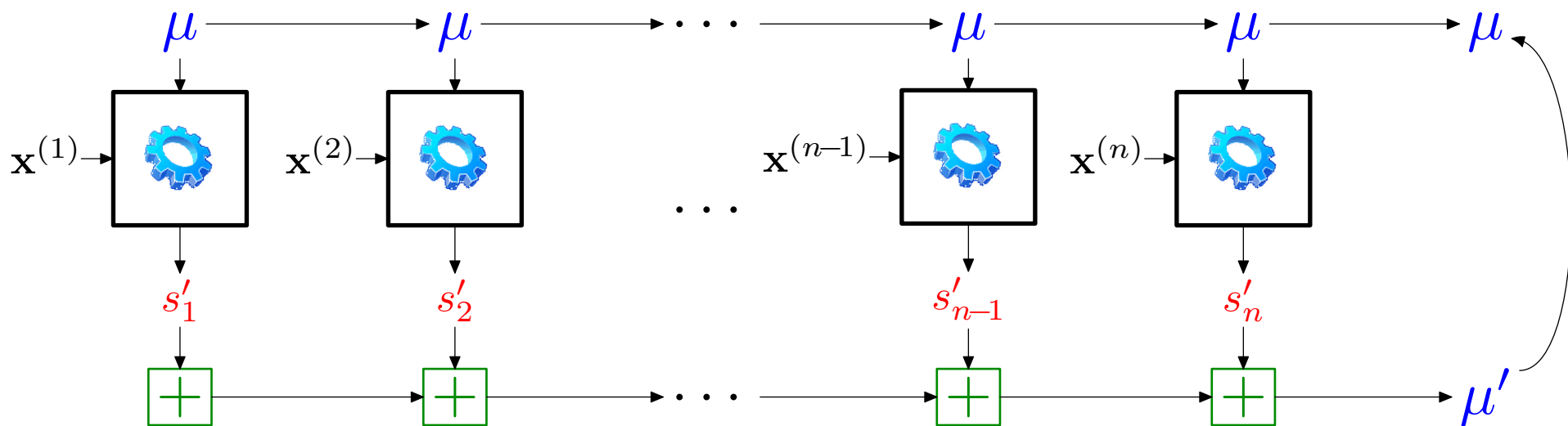
Data

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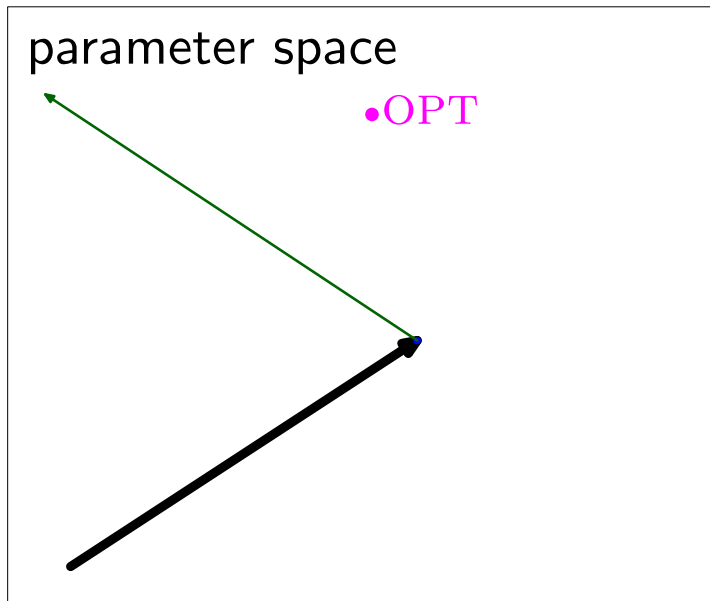




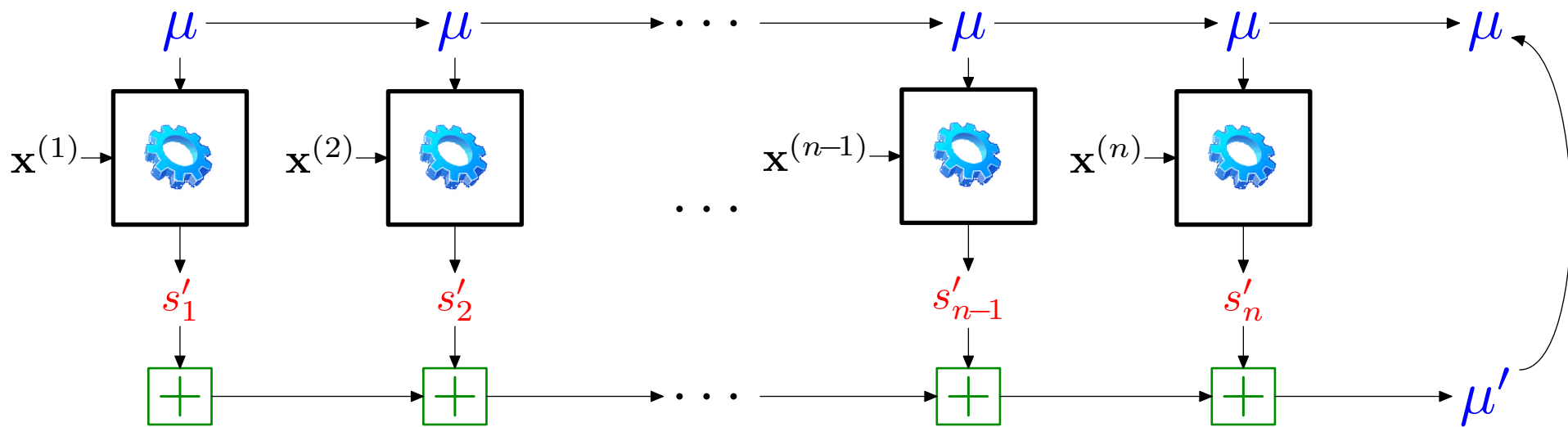
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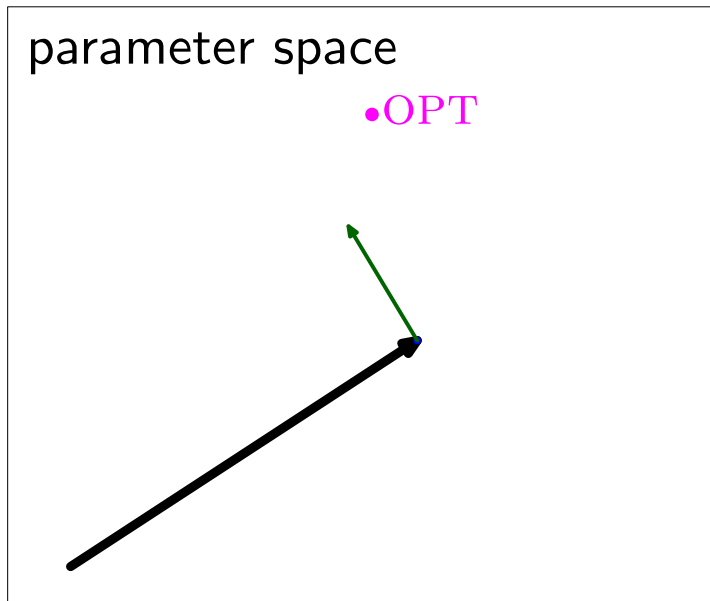
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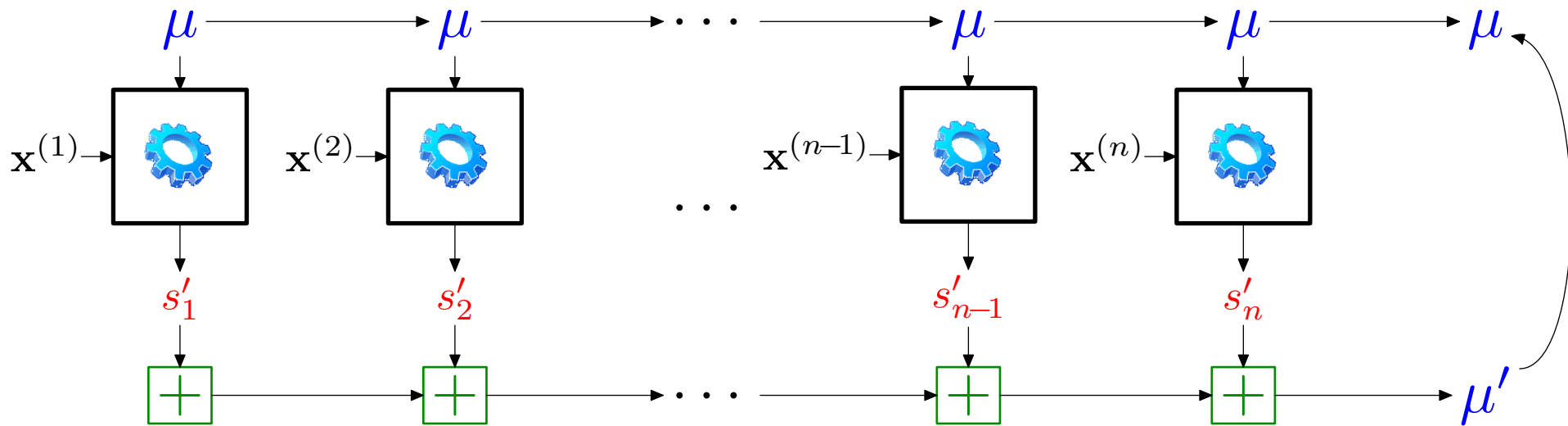


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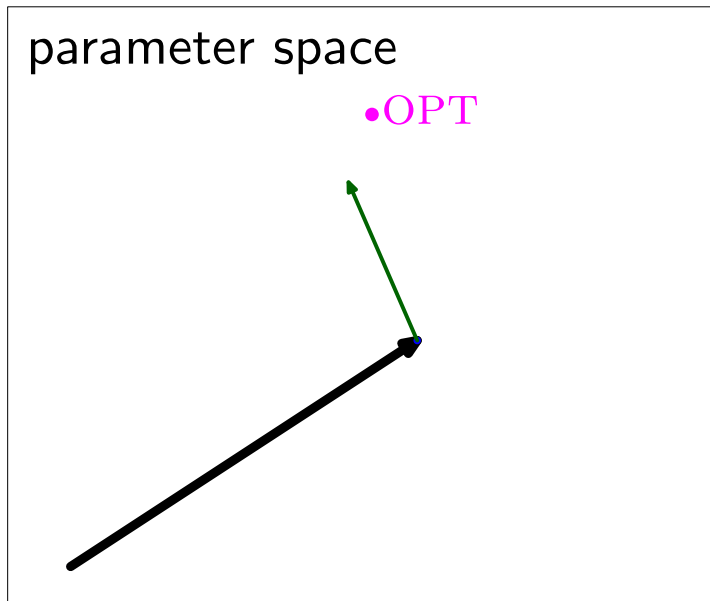


12 data points processed

# Batch EM

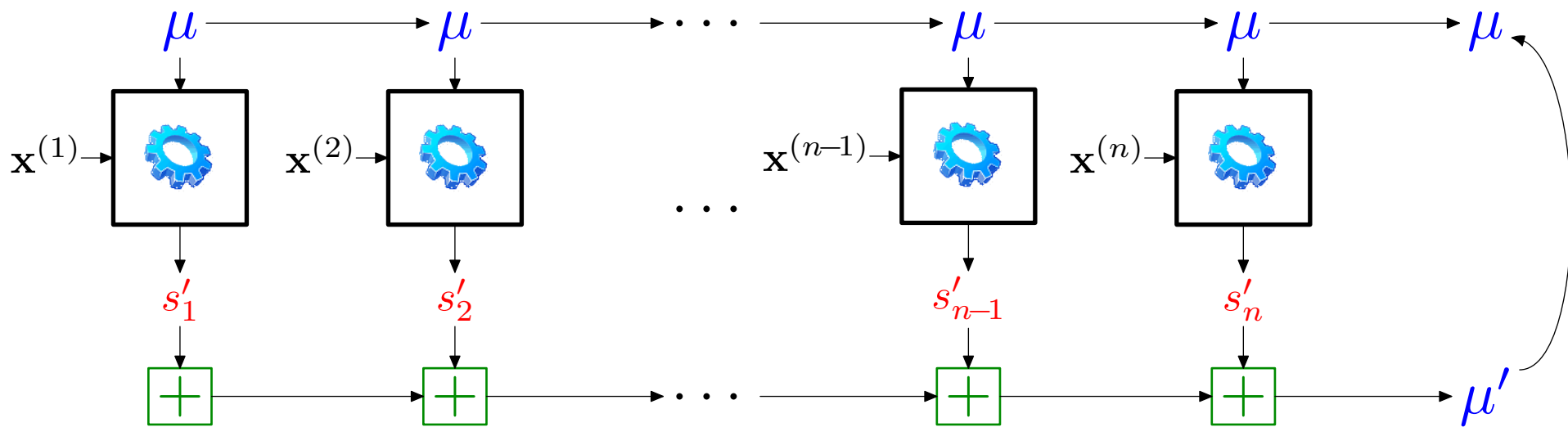


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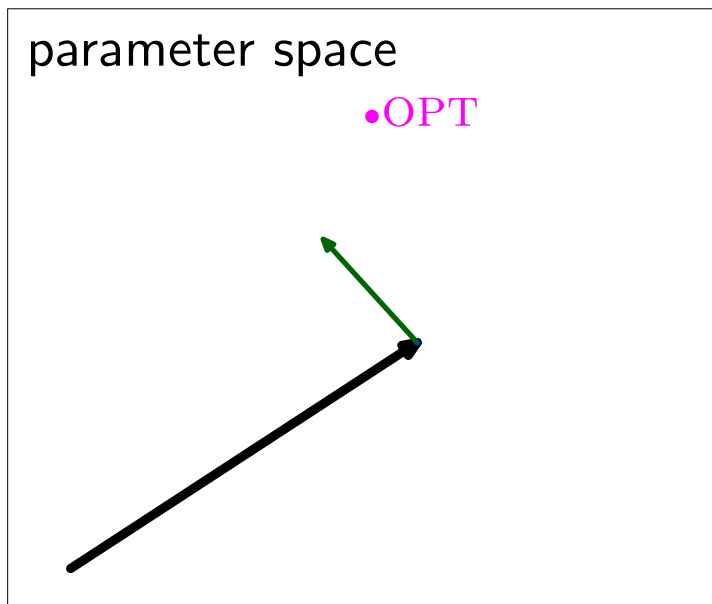


13 data points processed

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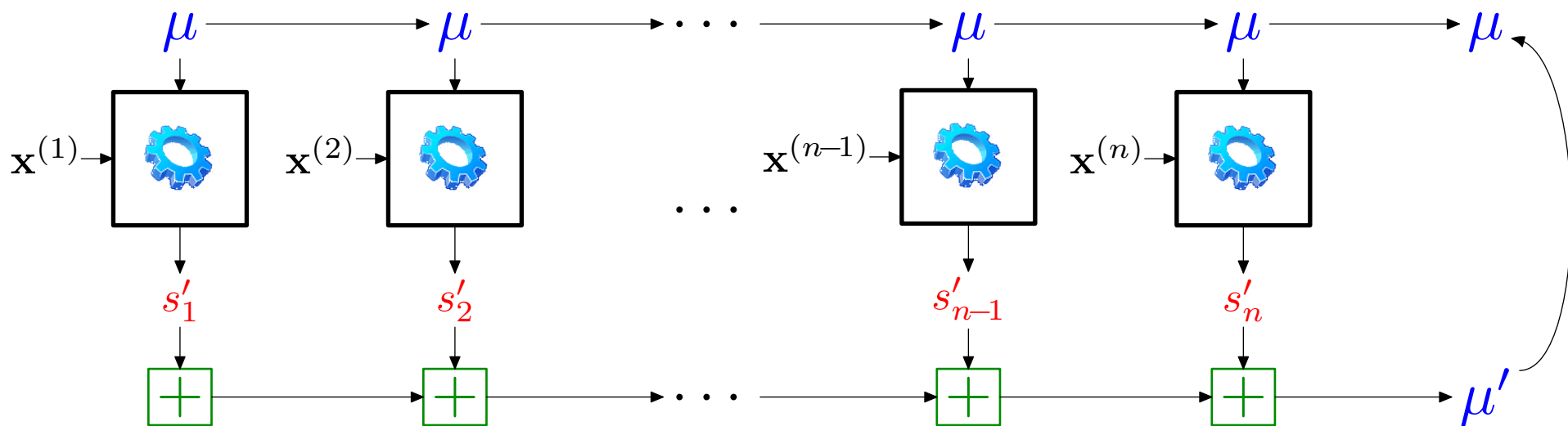


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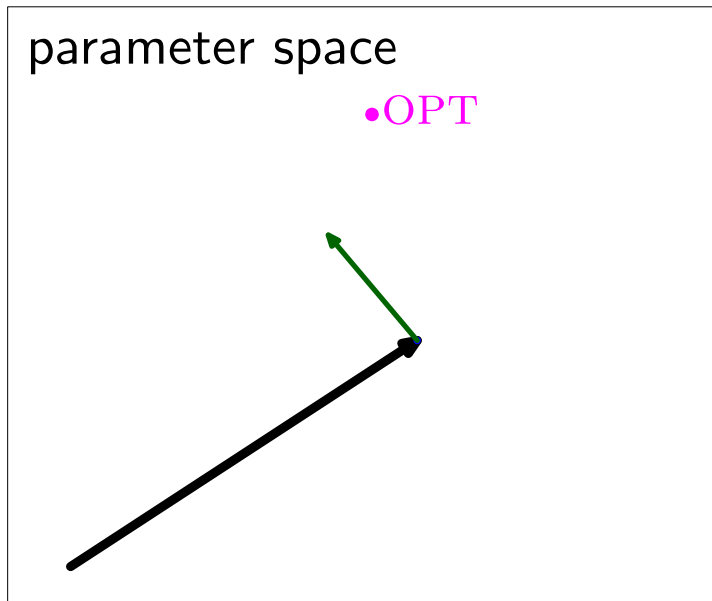
14 data points processed

# Batch EM



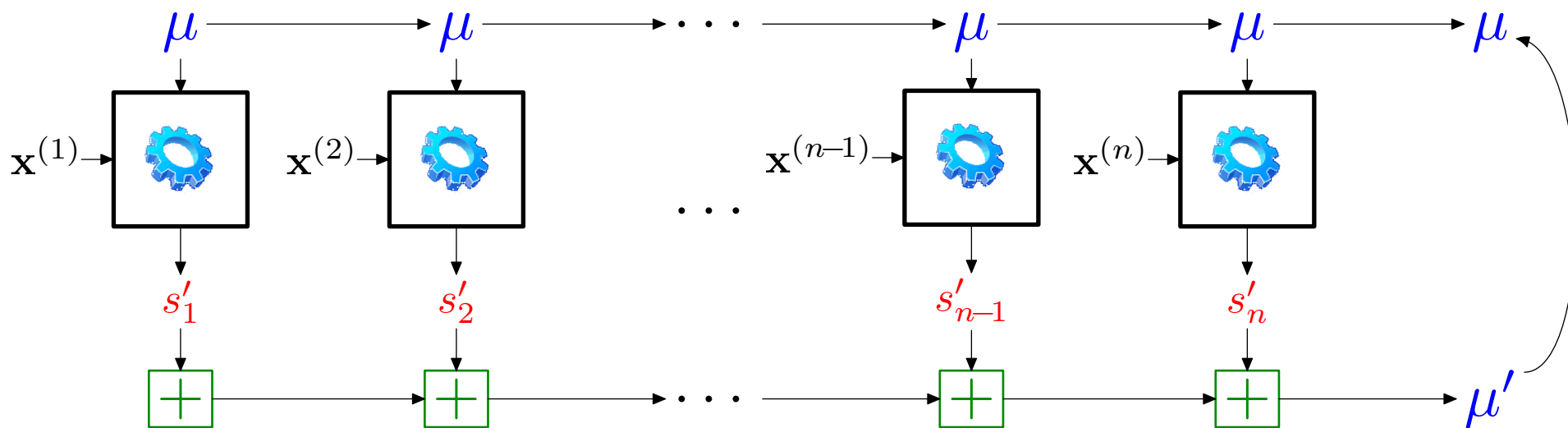
Data

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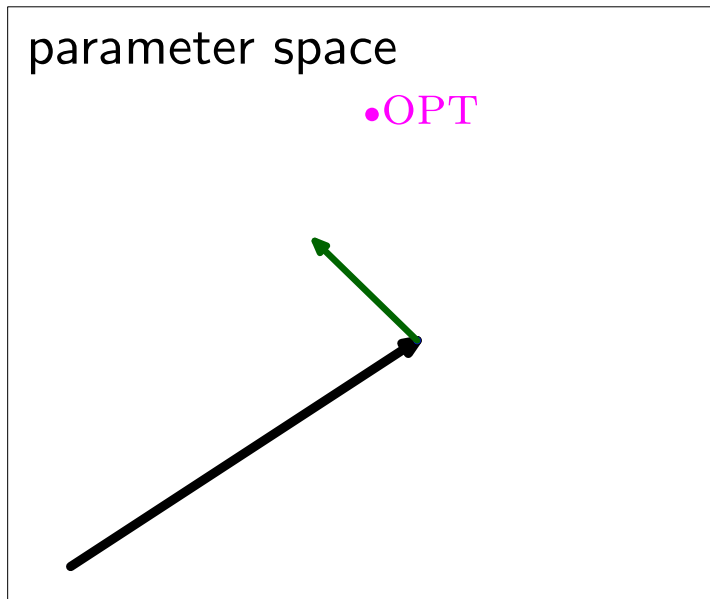


15 data points processed

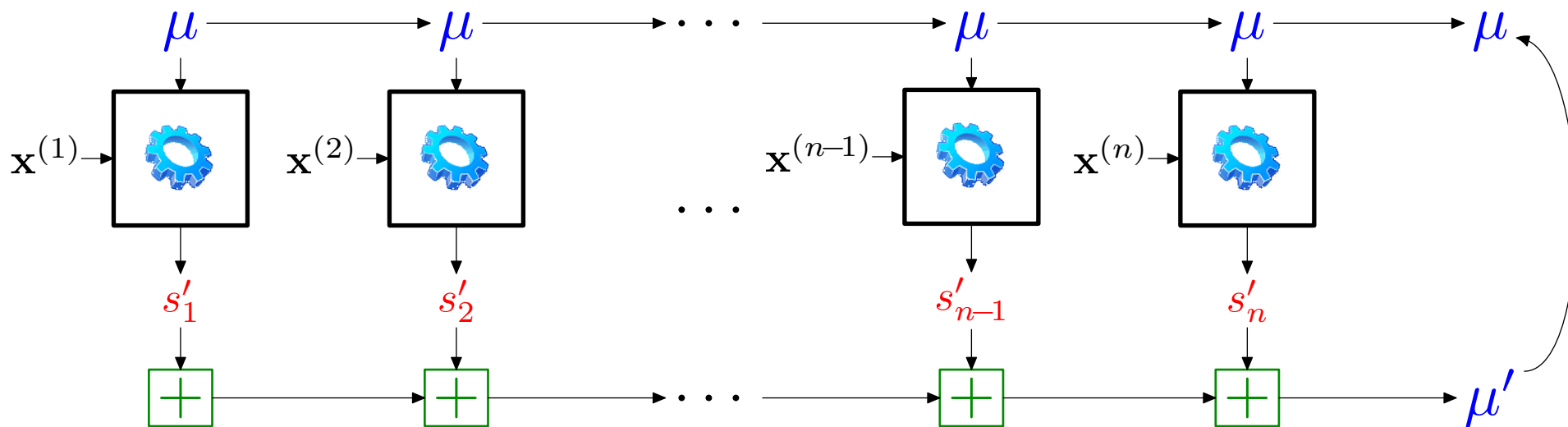
# Batch EM



Data  
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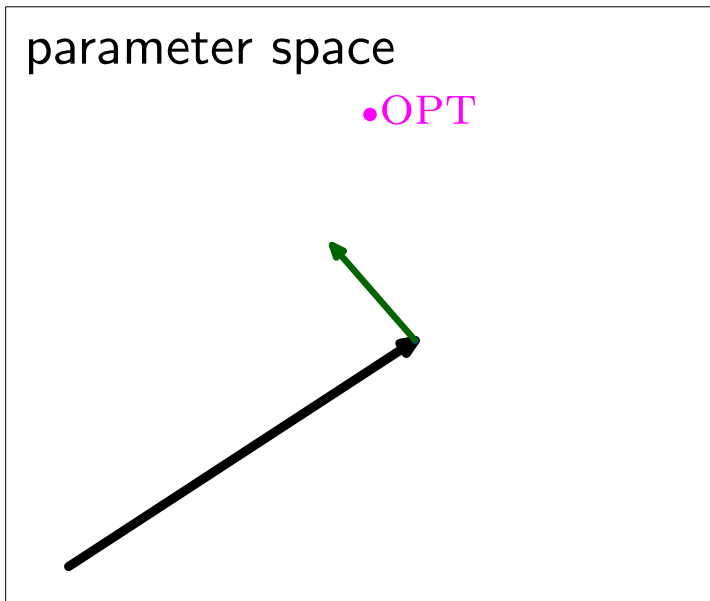


# Batch EM



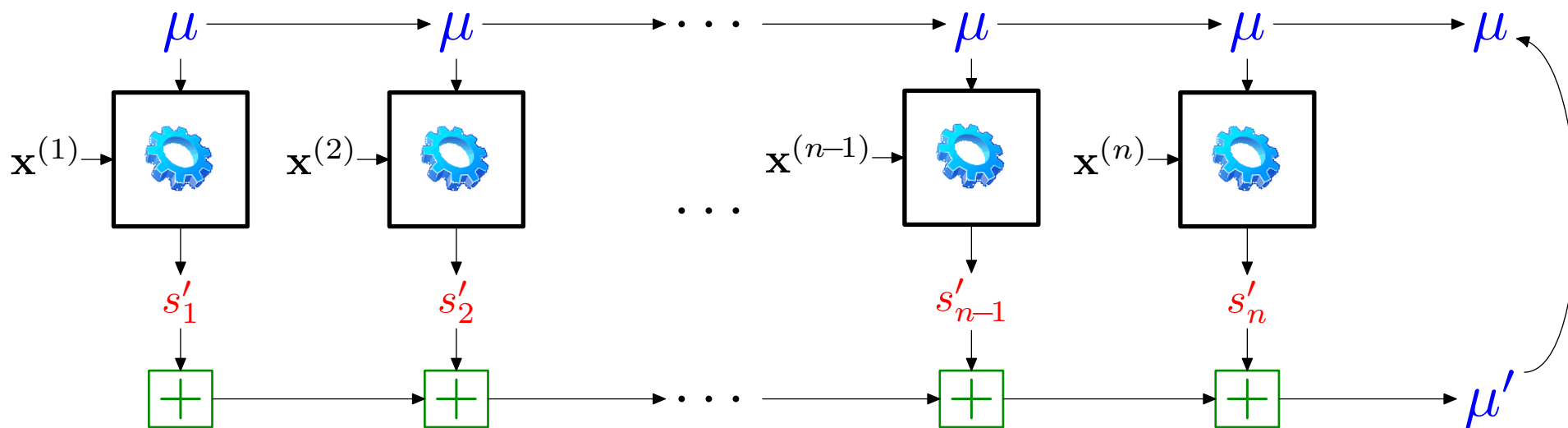
Data

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- (6,2)
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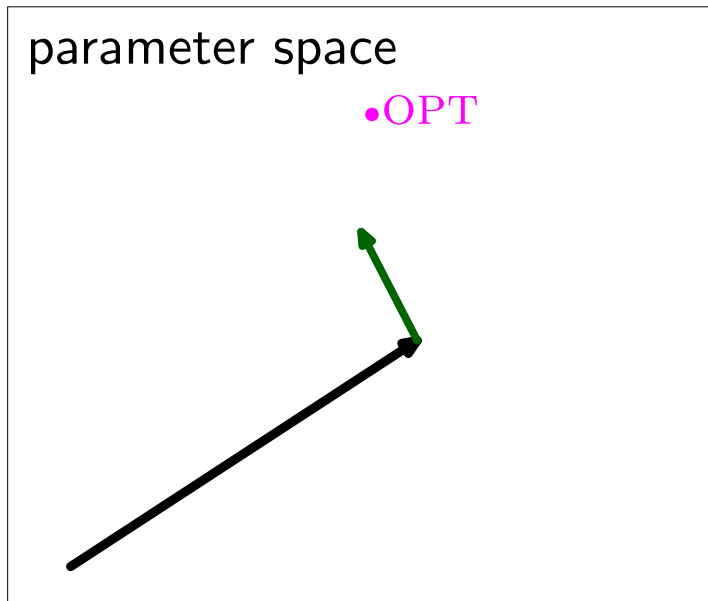
17 data points processed

# Batch EM



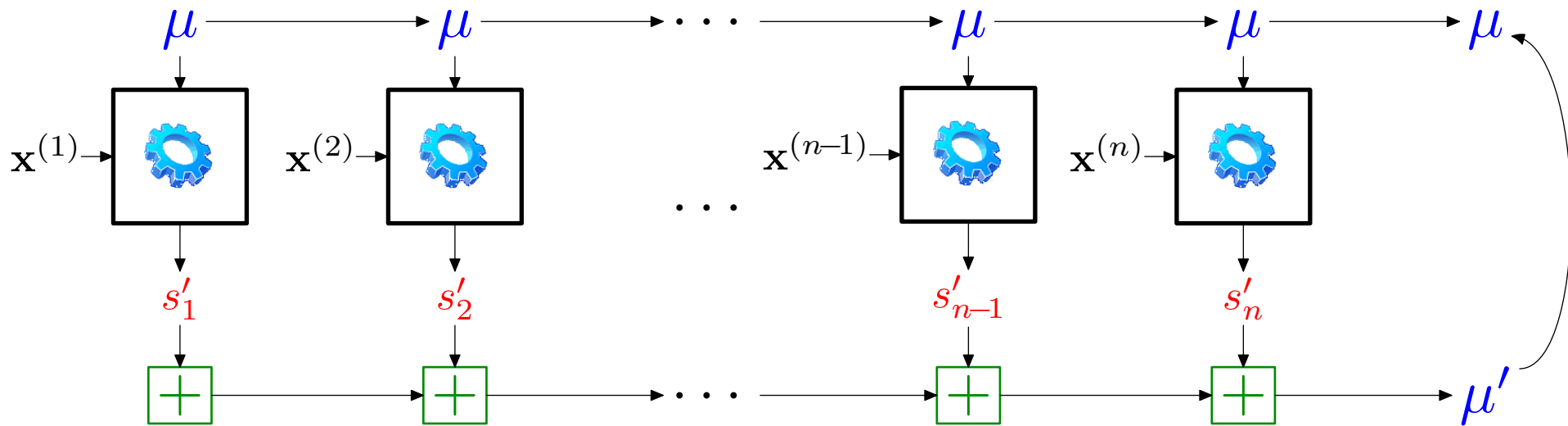
Data

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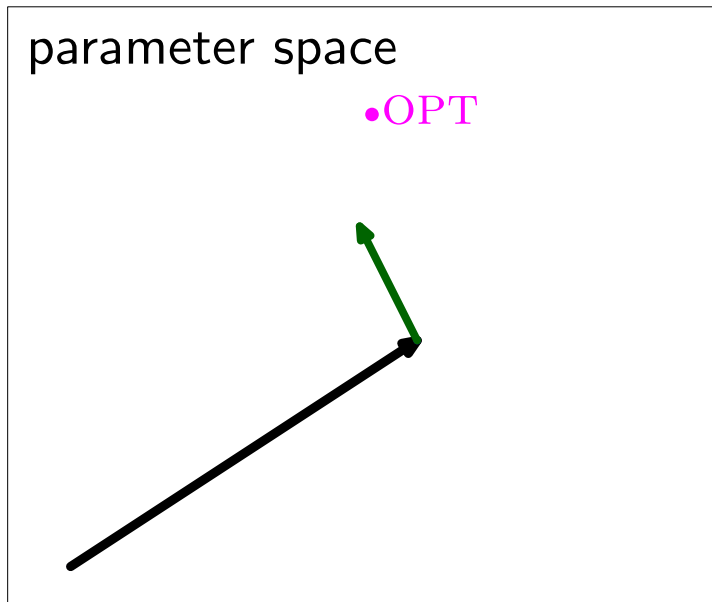


# Batch EM

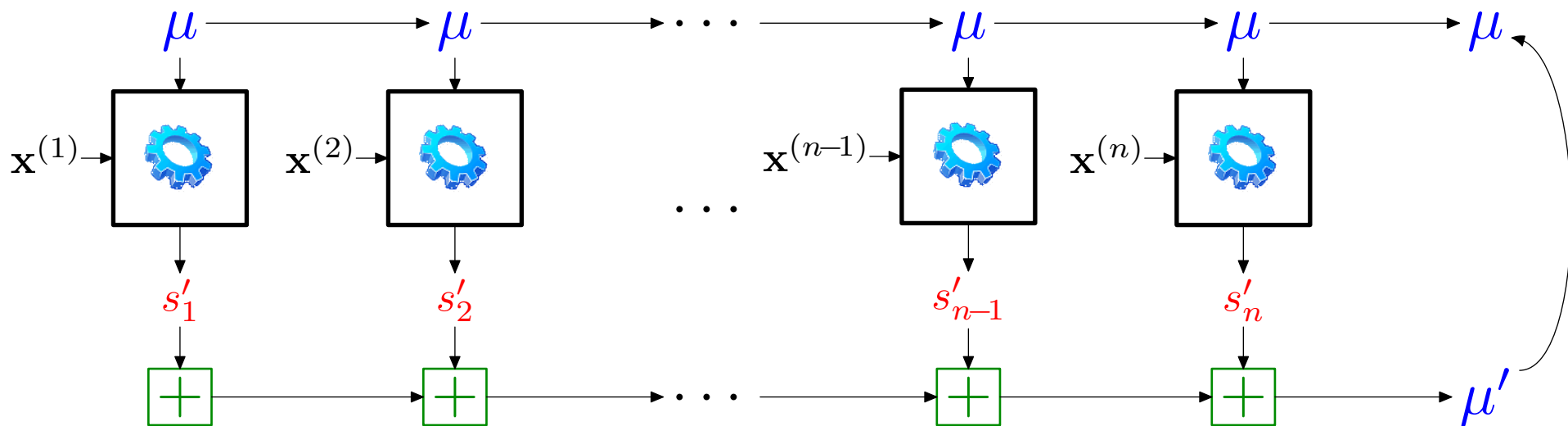


Data

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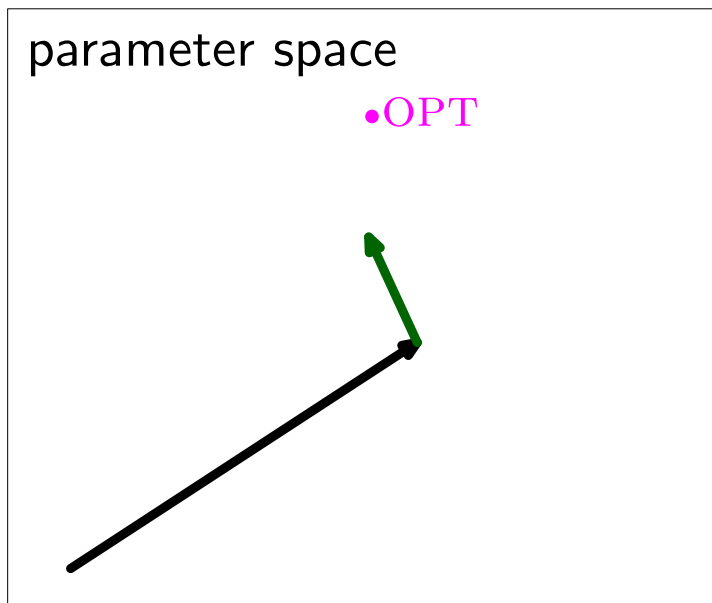


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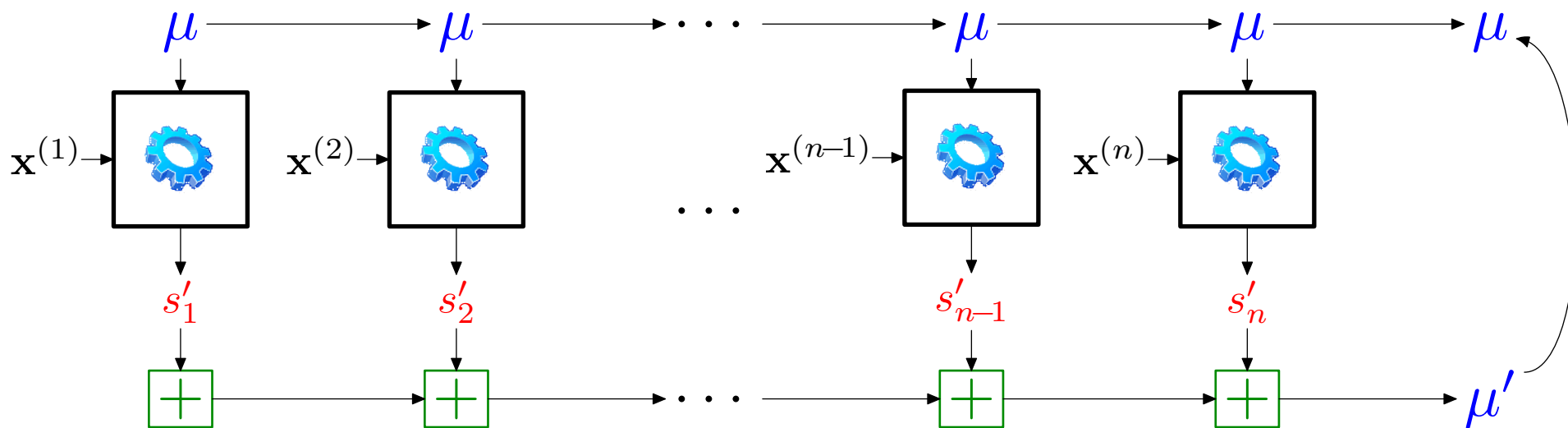


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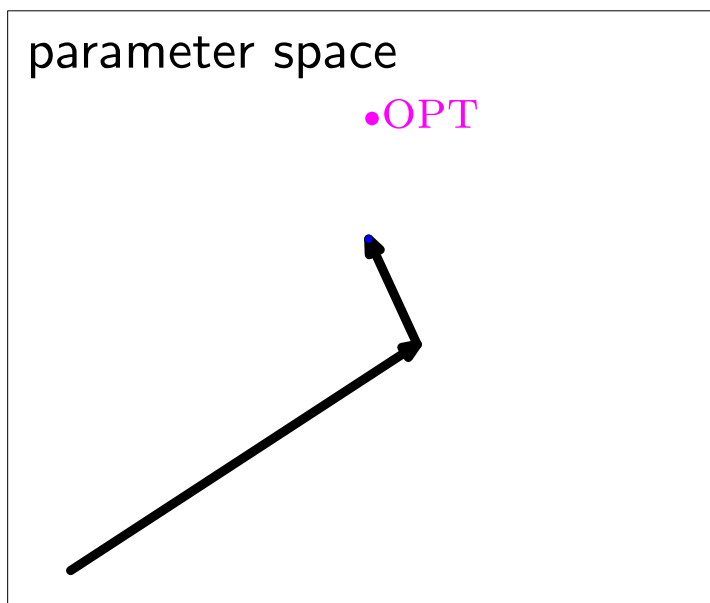


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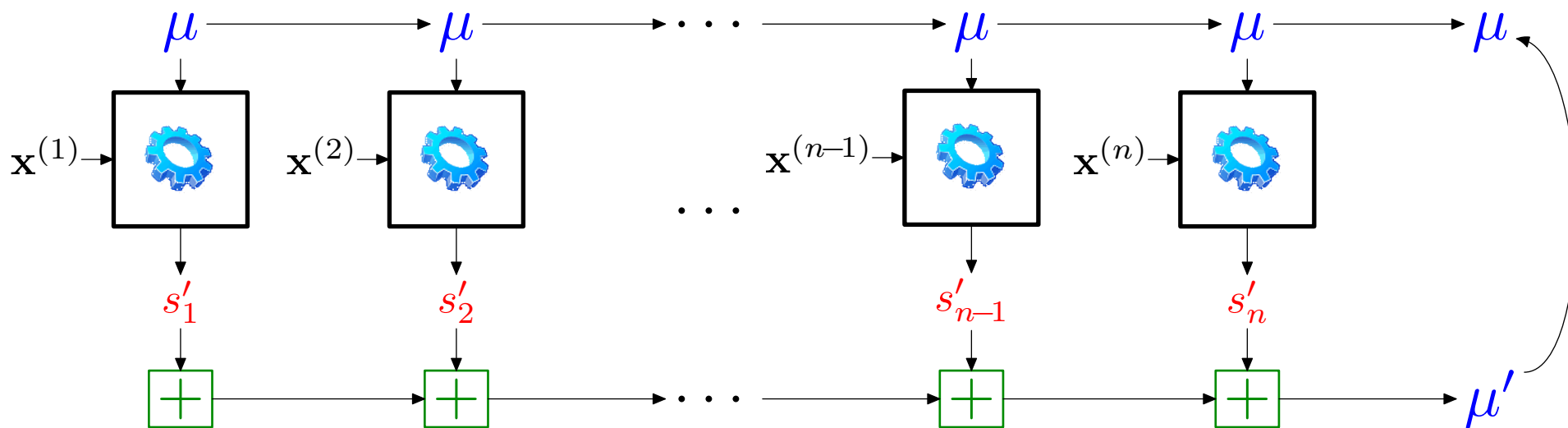
Data

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- (3,6)
- (4,3)



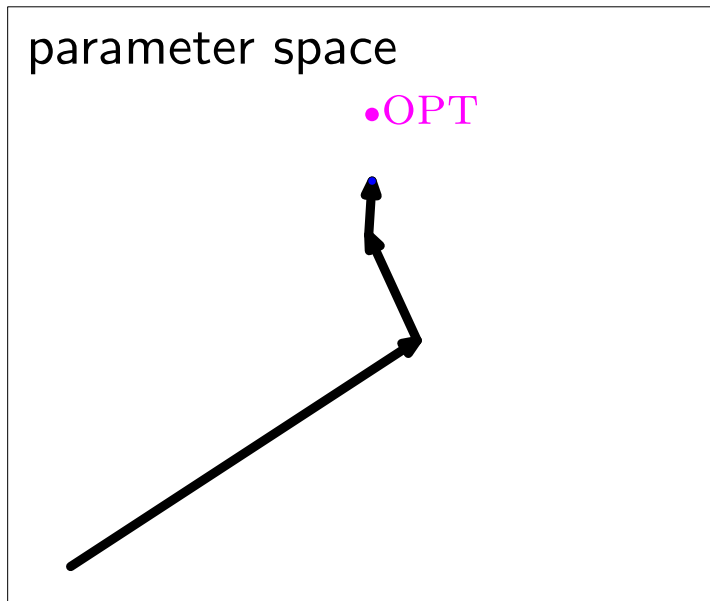
20 data points processed

# Batch EM



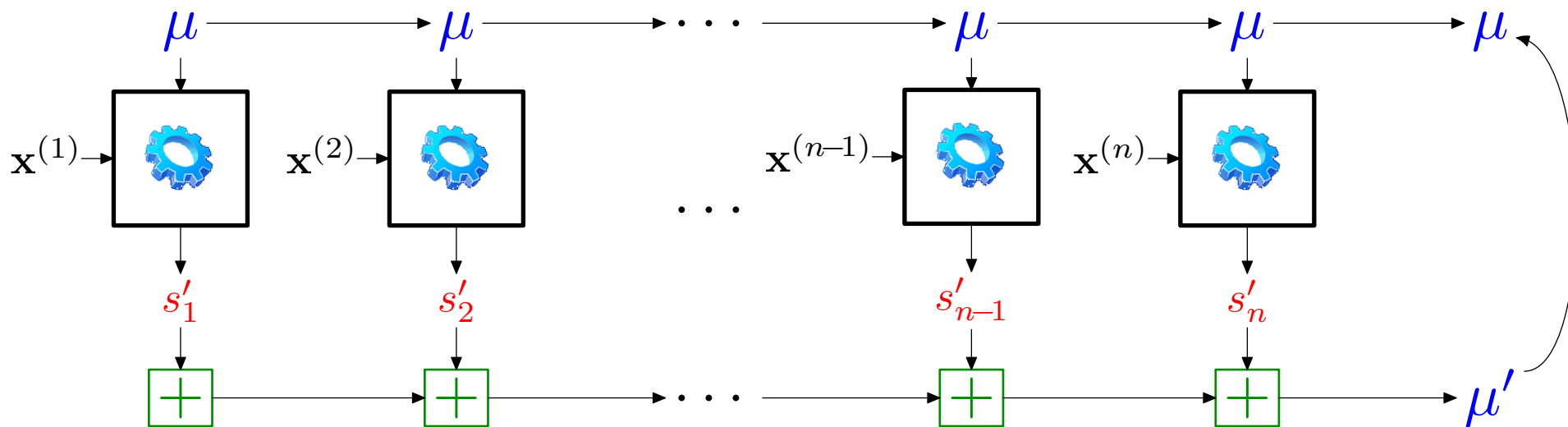
Data

- (0,8)
- (6,2)
- (3,8)
- (2,1)
- (3,5)
- (2,4)
- (4,4)
- (5,7)
- (3,6)
- (4,3)



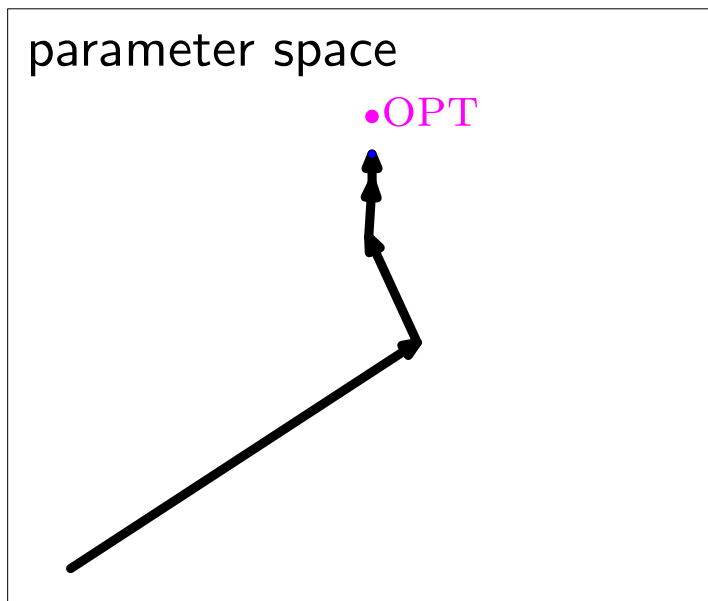
30 data points processed

# Batch EM

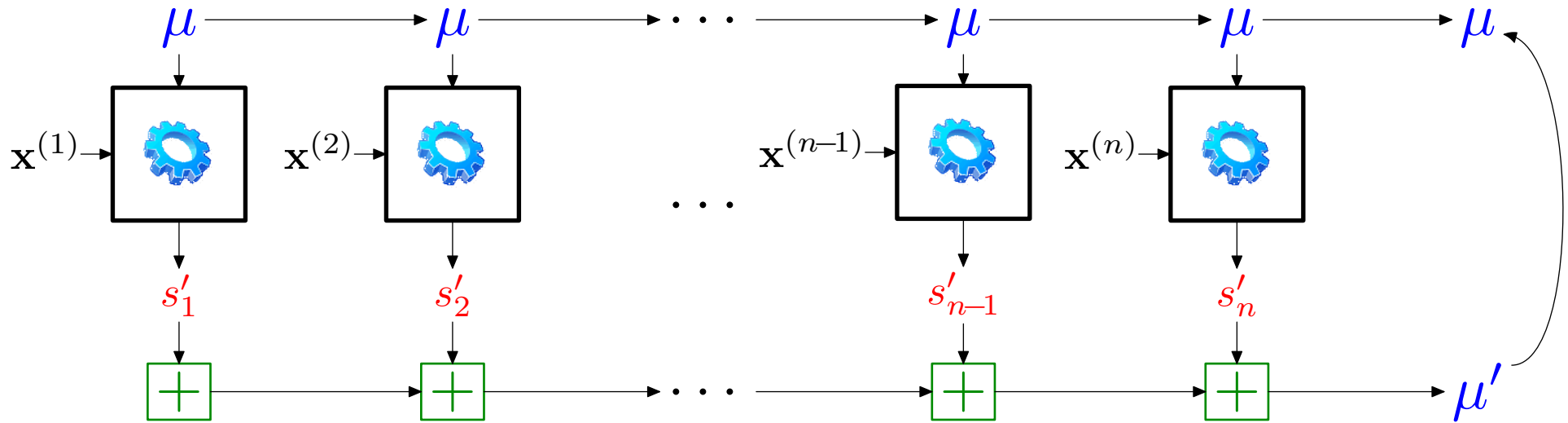


Data

- (0,8)
- (6,2)
- (3,8)
- (2,1)
- (3,5)
- (2,4)
- (4,4)
- (5,7)
- (3,6)
- (4,3)



# Batch EM



Data

(0,8)  
(6,2)  
(3,8)  
(2,1)  
(3,5)  
(2,4)  
(4,4)  
(5,7)  
(3,6)  
(4,3)

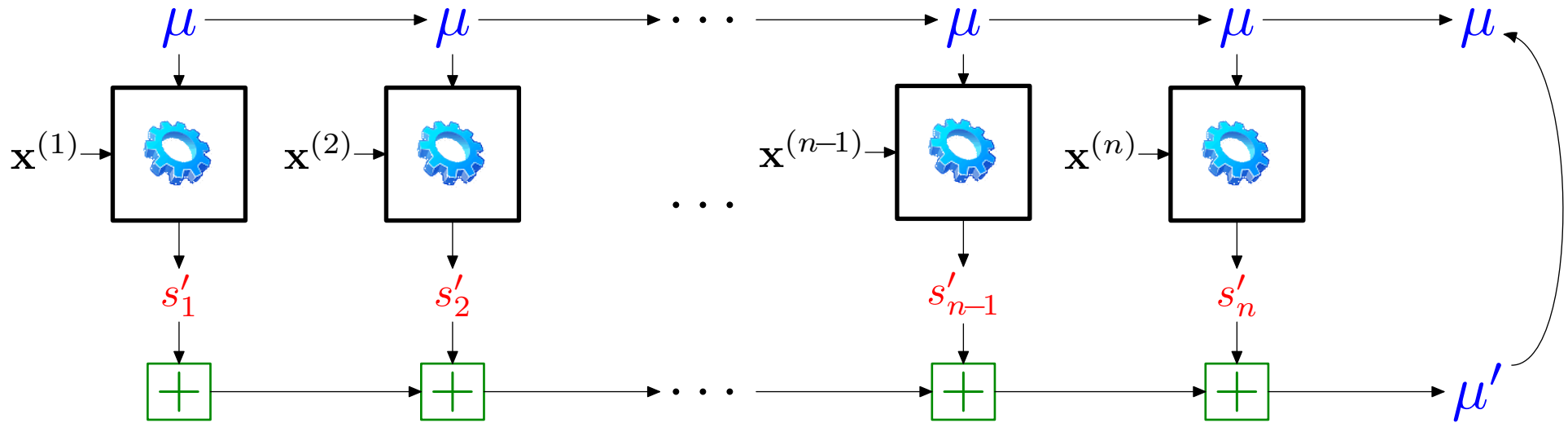
parameter space

•OPT

40 data points processed

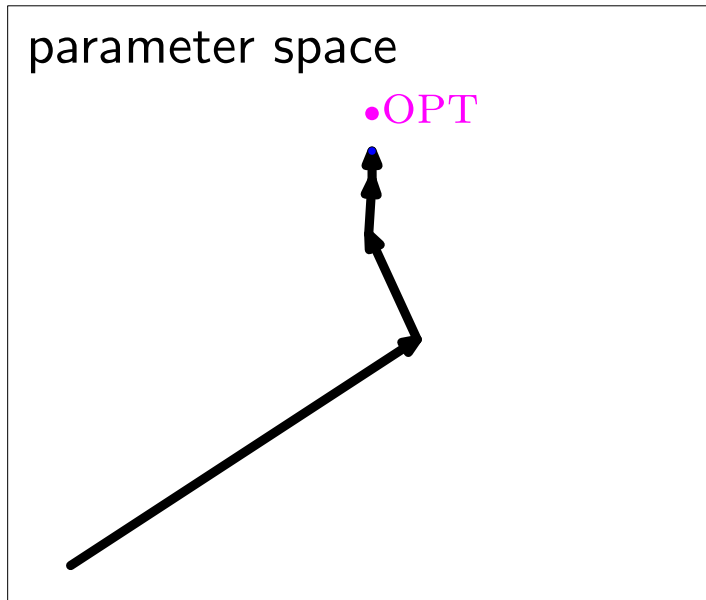
- Spend a lot of time computing new parameters exactly, but have rough estimate much earlier

# Batch EM



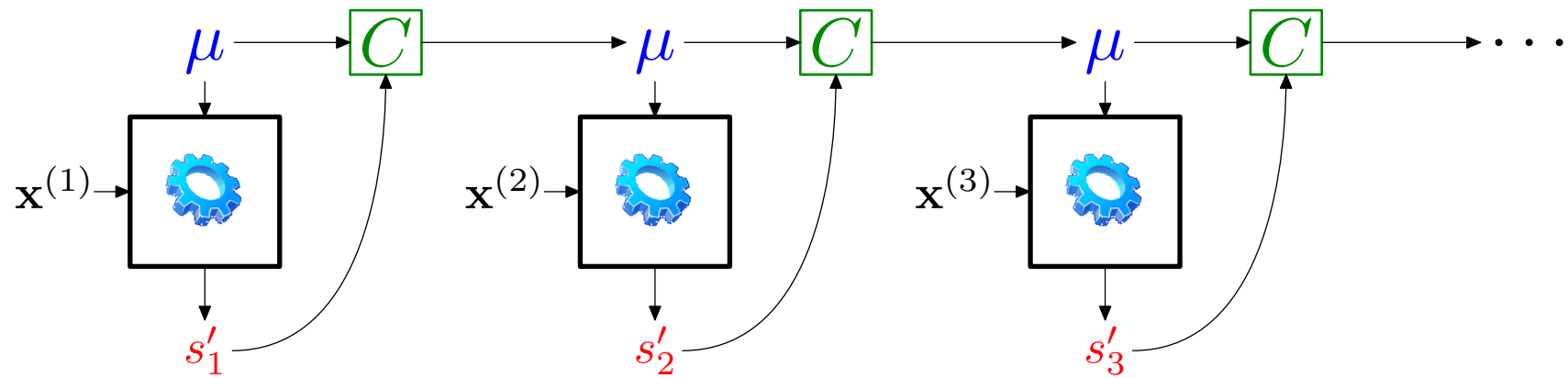
Data

(0,8)  
(6,2)  
(3,8)  
(2,1)  
(3,5)  
(2,4)  
(4,4)  
(5,7)  
(3,6)  
(4,3)



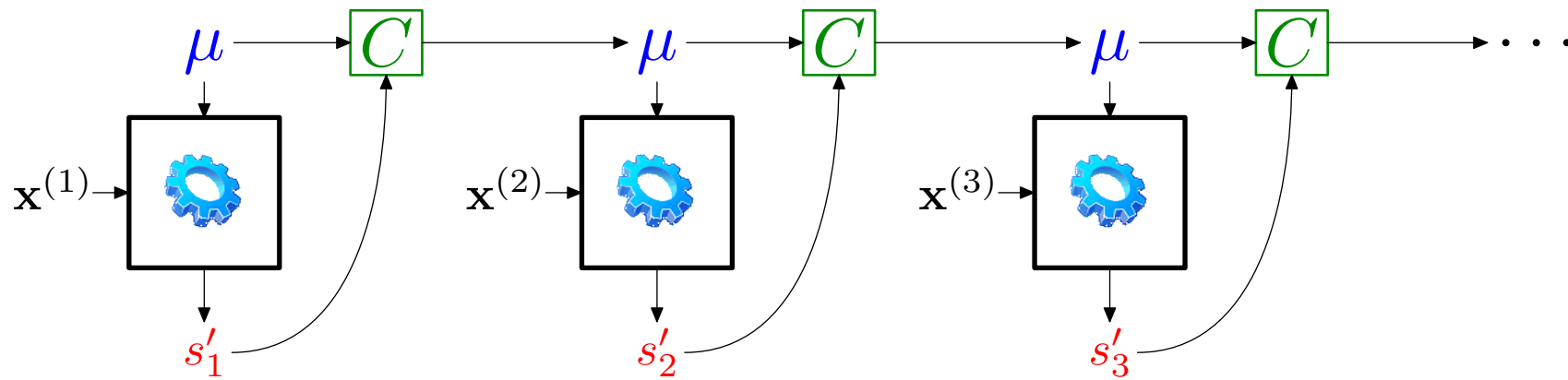
- Spend a lot of time computing new parameters exactly, but have rough estimate much earlier
- New parameters are intermediate, so don't need to obsess about the exact value

# Online EM [Cappé & Moulines, 2009]

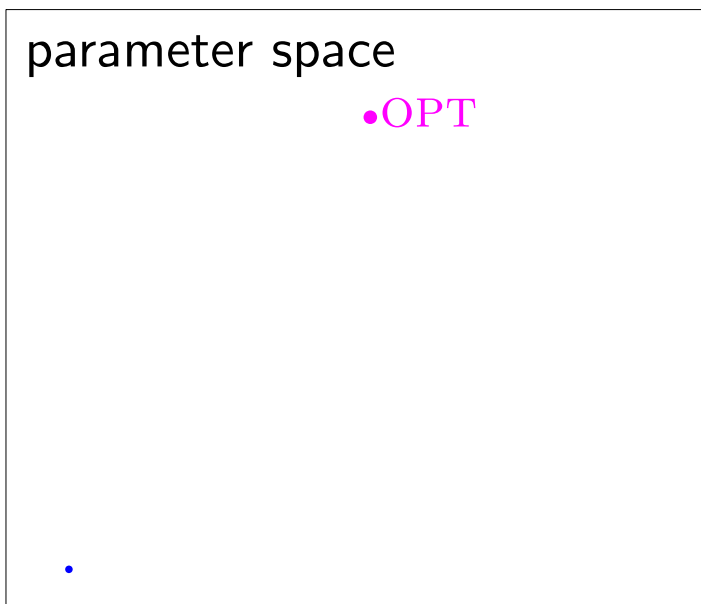




# Online EM [Cappé & Moulines, 2009]

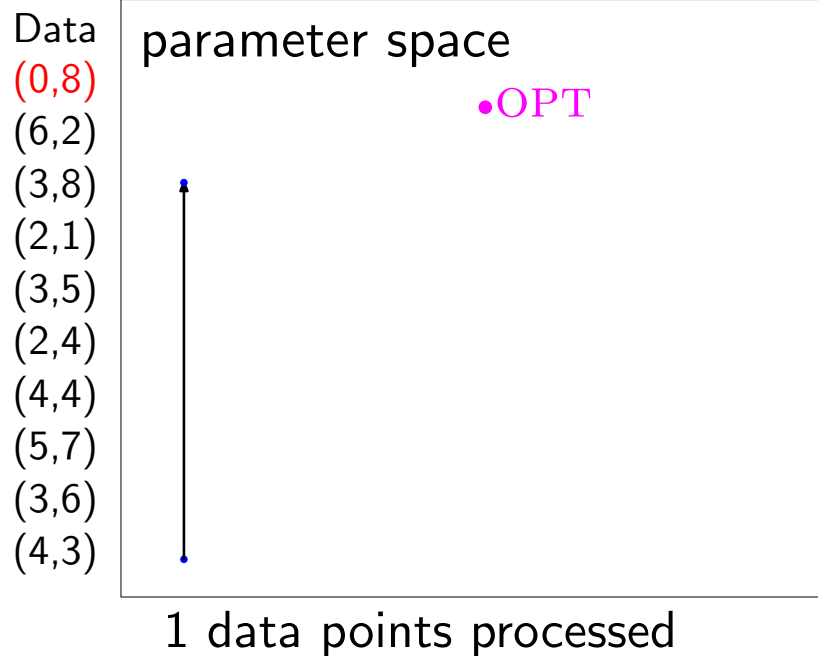
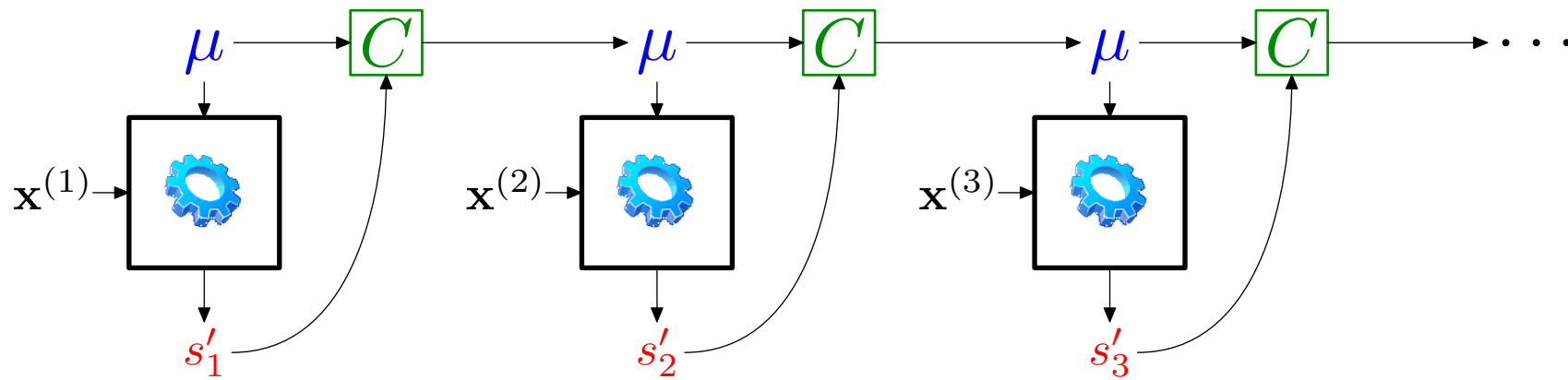


Data  
(0,8)  
(6,2)  
(3,8)  
(2,1)  
(3,5)  
(2,4)  
(4,4)  
(5,7)  
(3,6)  
(4,3)

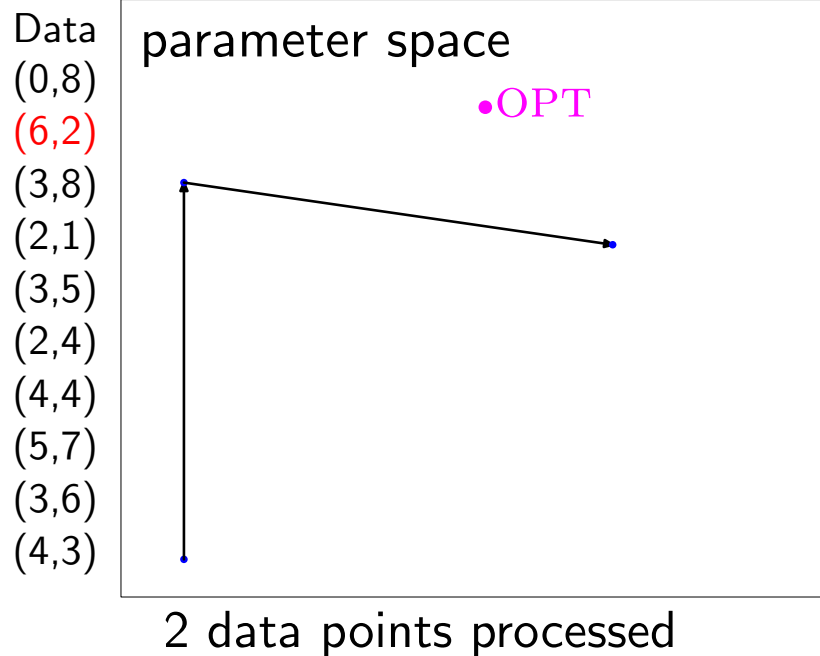
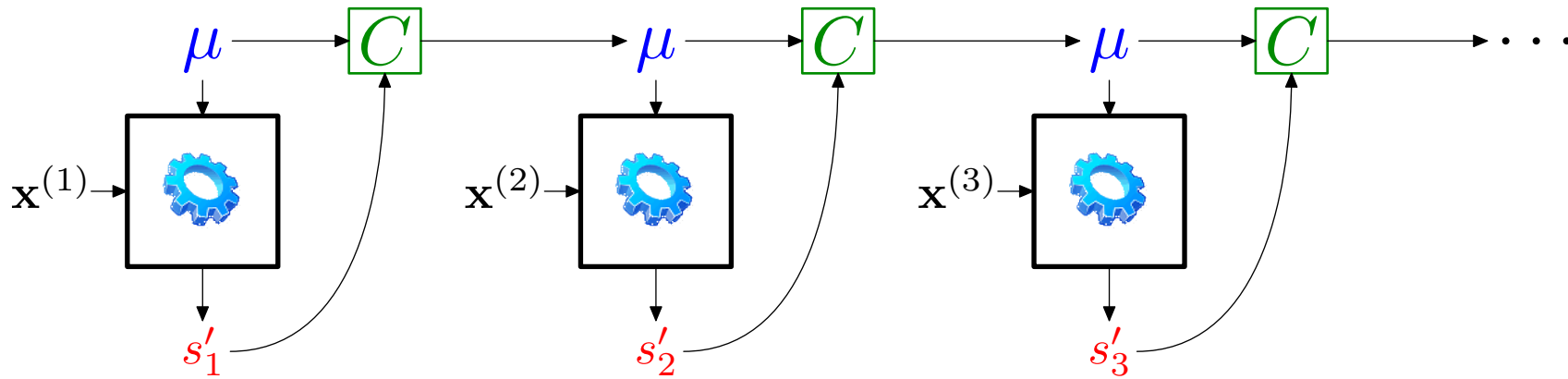


0 data points processed

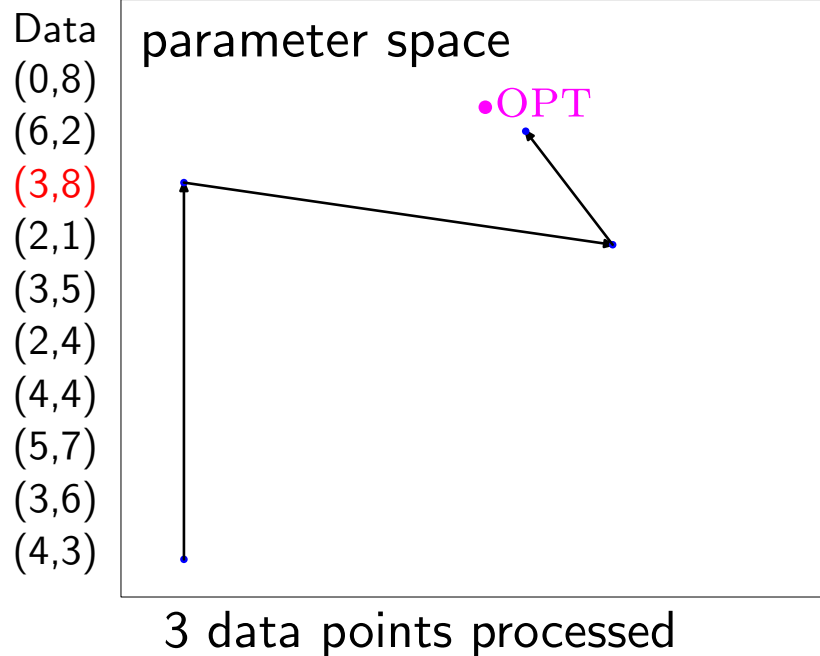
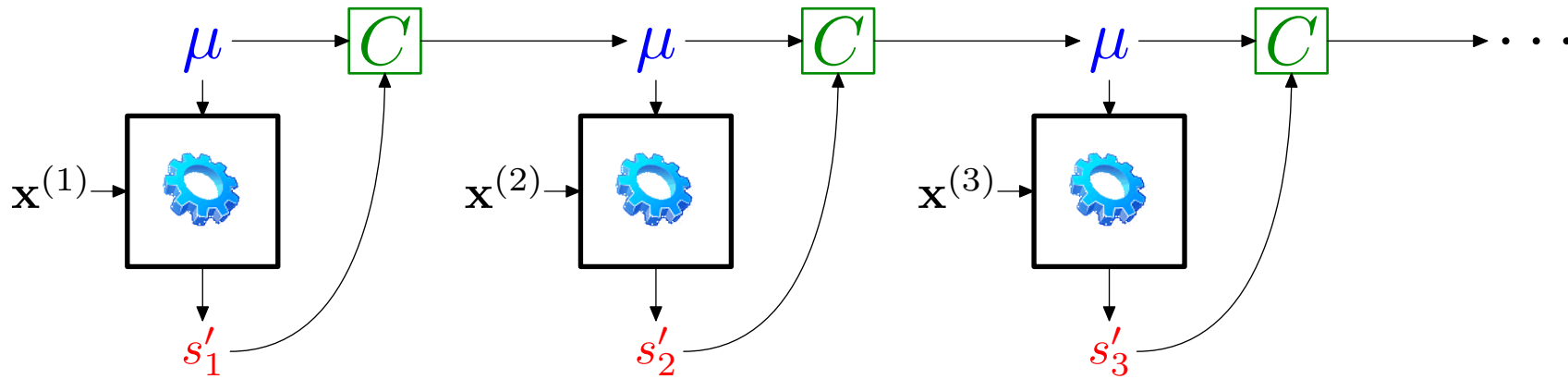
# Online EM [Cappé & Moulines, 2009]



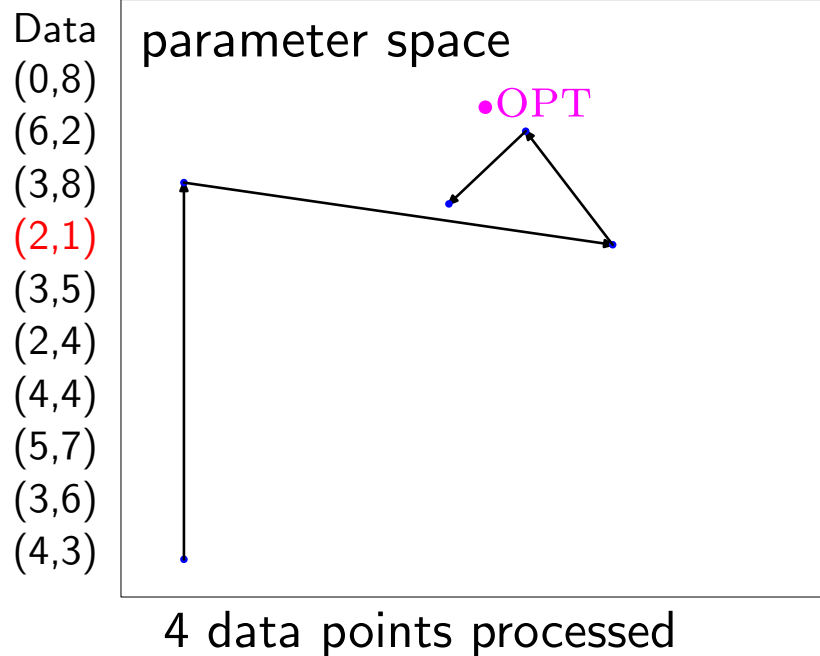
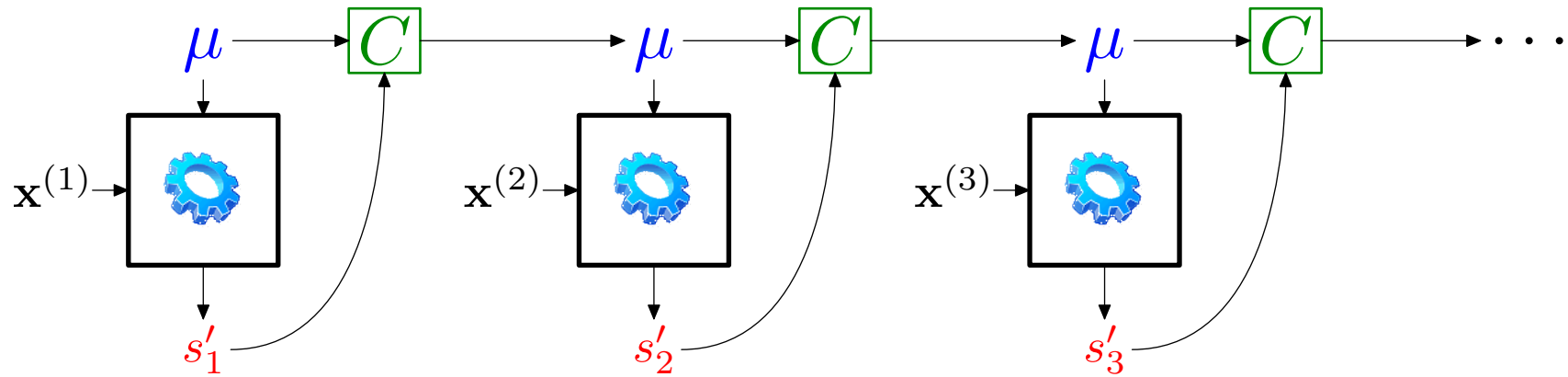
# Online EM [Cappé & Moulines, 2009]



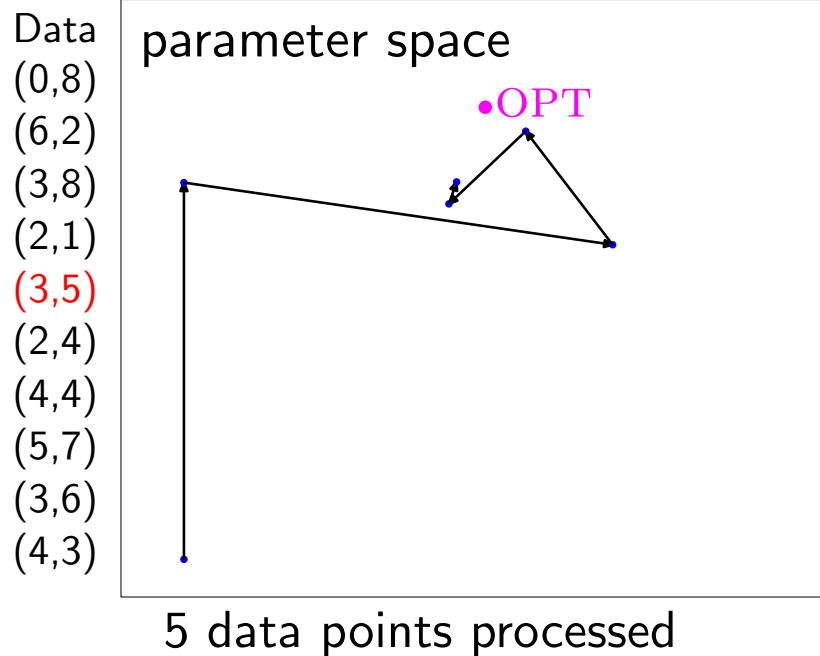
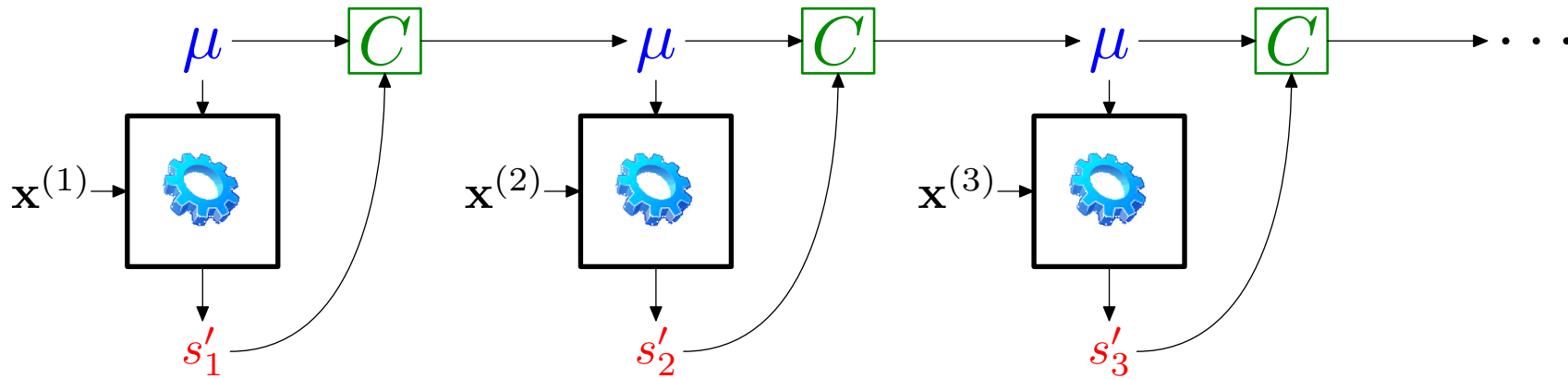
# Online EM [Cappé & Moulines, 2009]



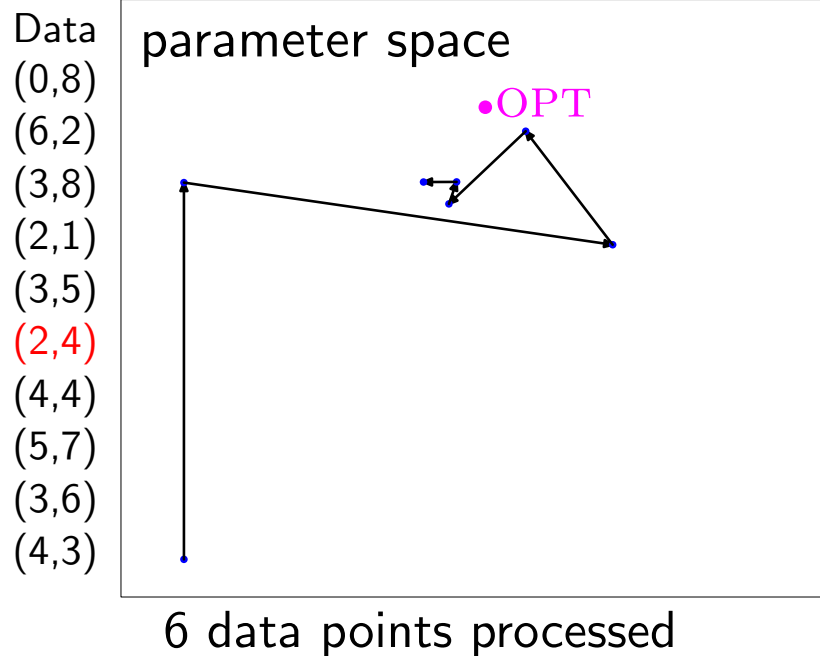
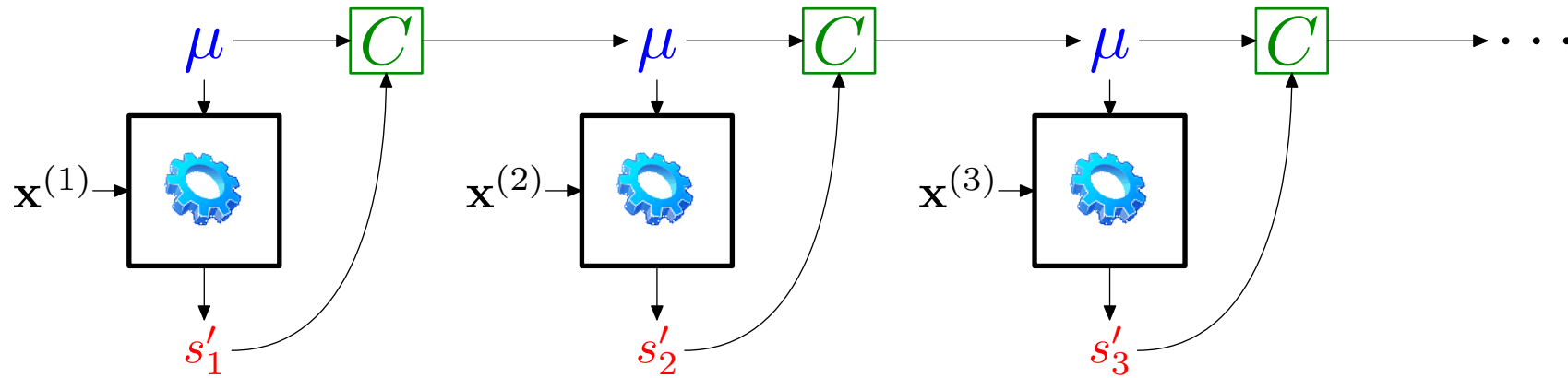
# Online EM [Cappé & Moulines, 2009]



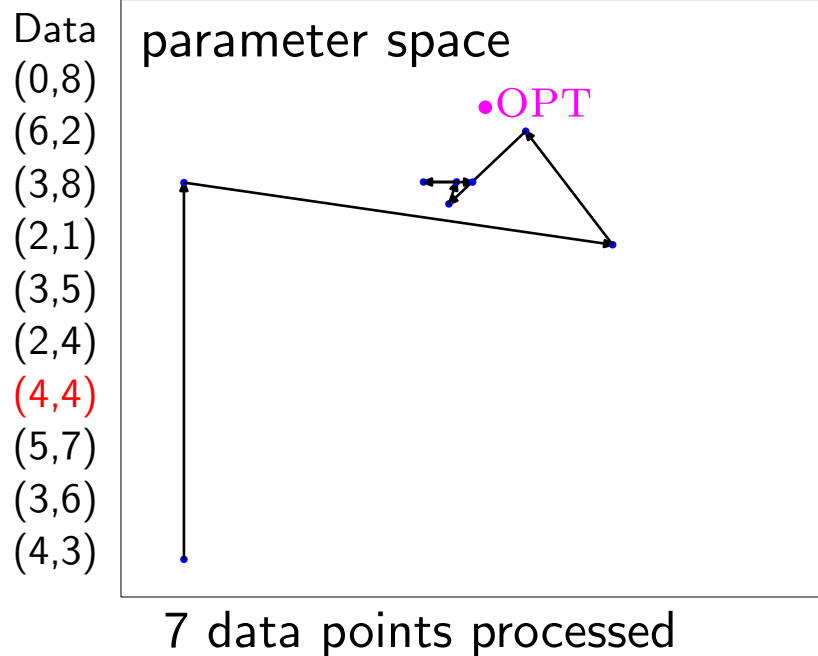
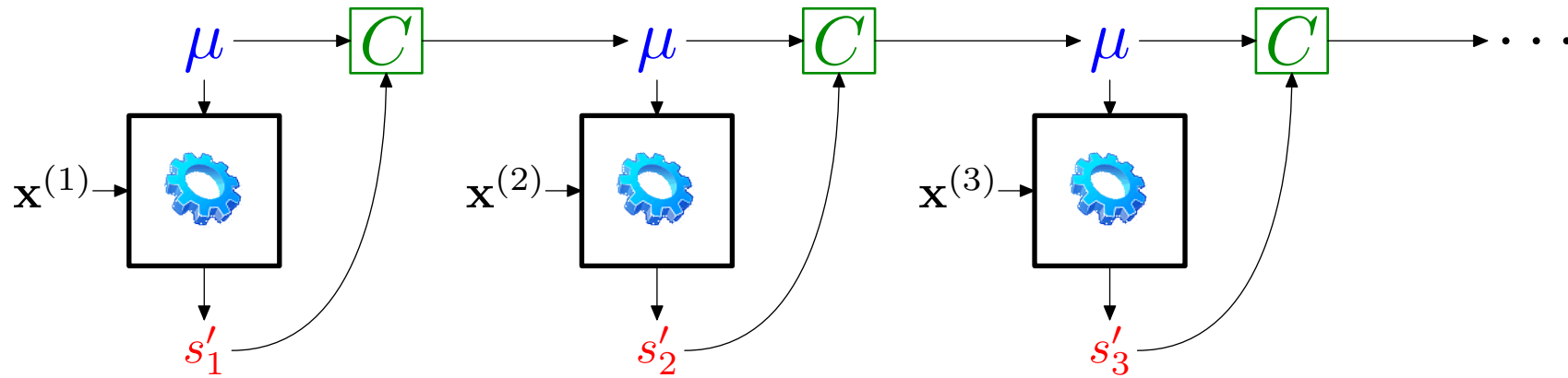
# Online EM [Cappé & Moulines, 2009]



# Online EM [Cappé & Moulines, 2009]

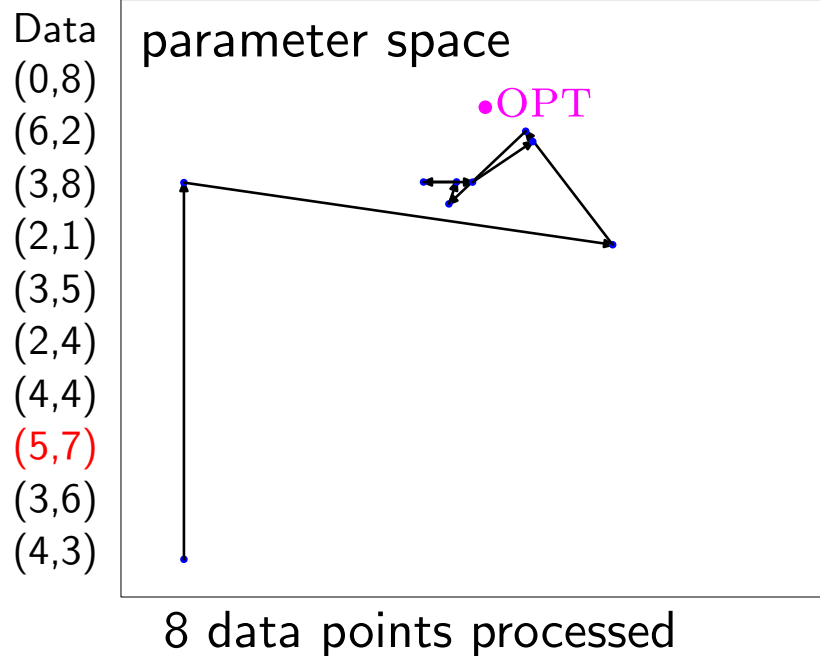
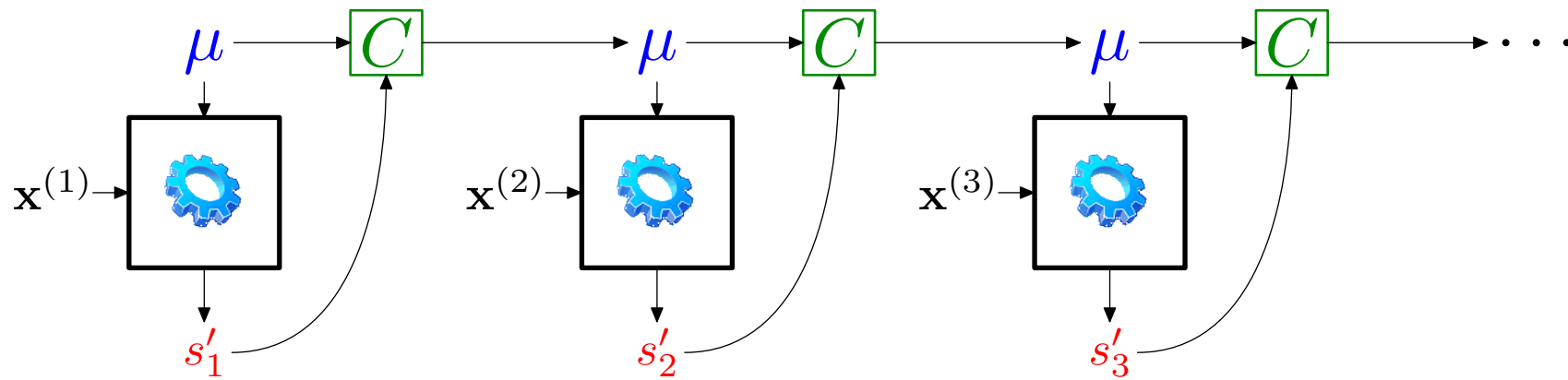


# Online EM [Cappé & Moulines, 2009]

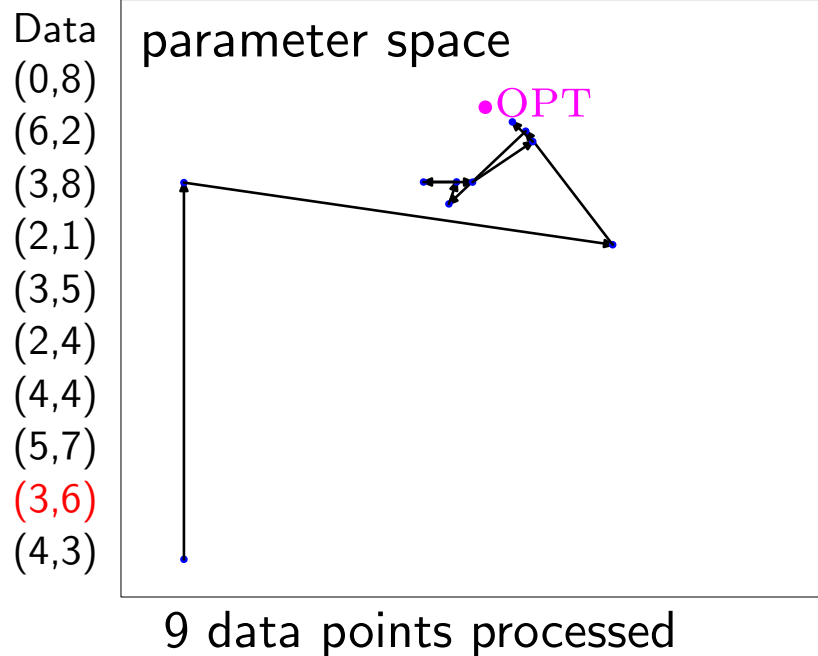
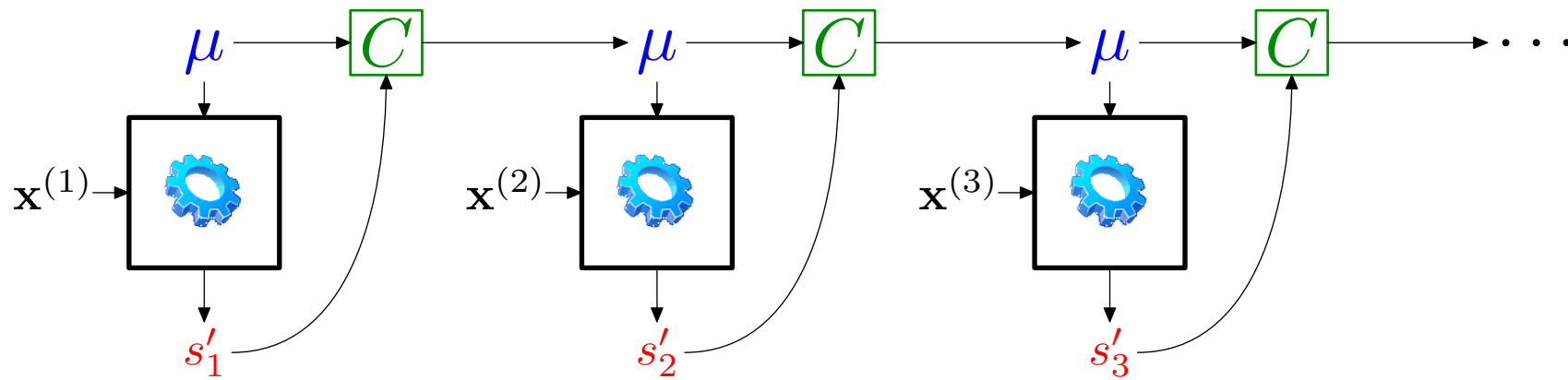




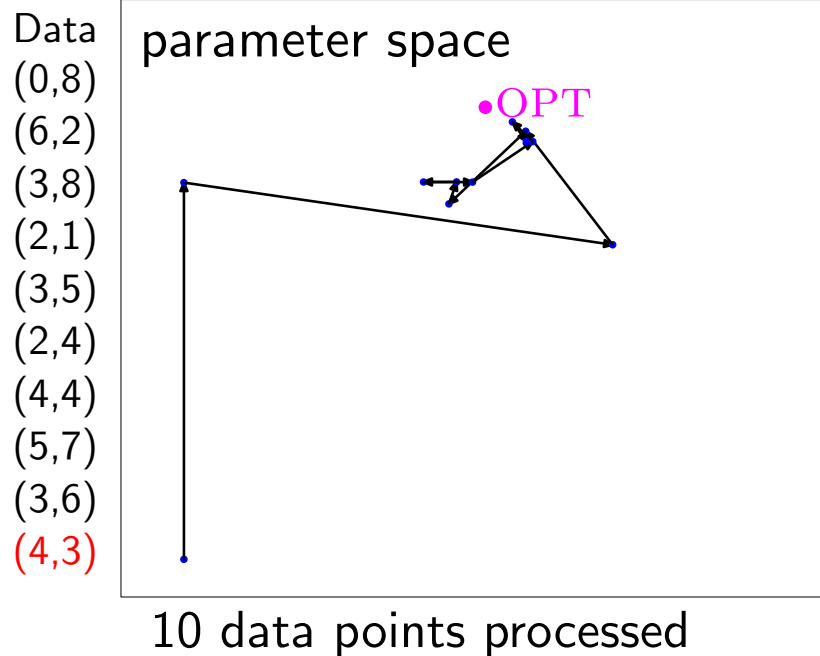
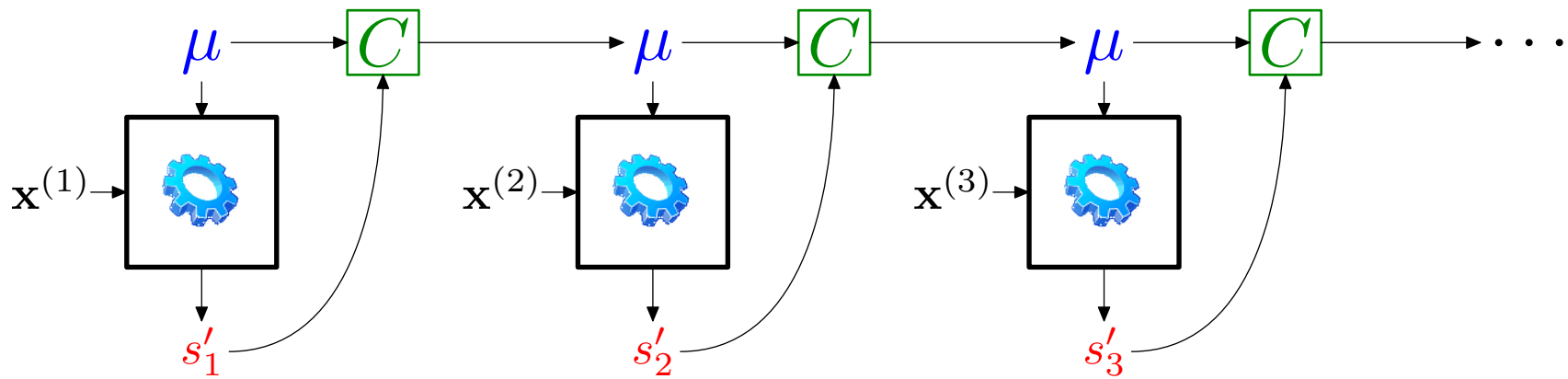
# Online EM [Cappé & Moulines, 2009]



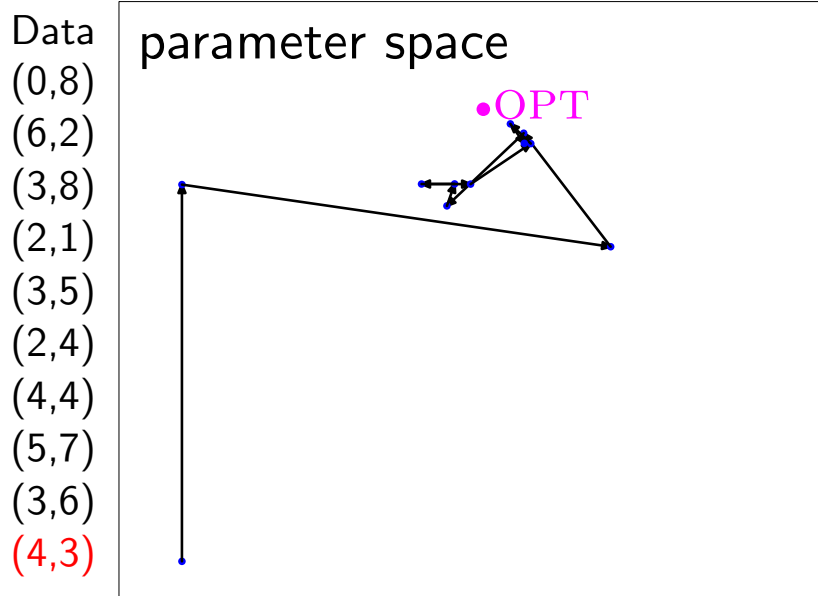
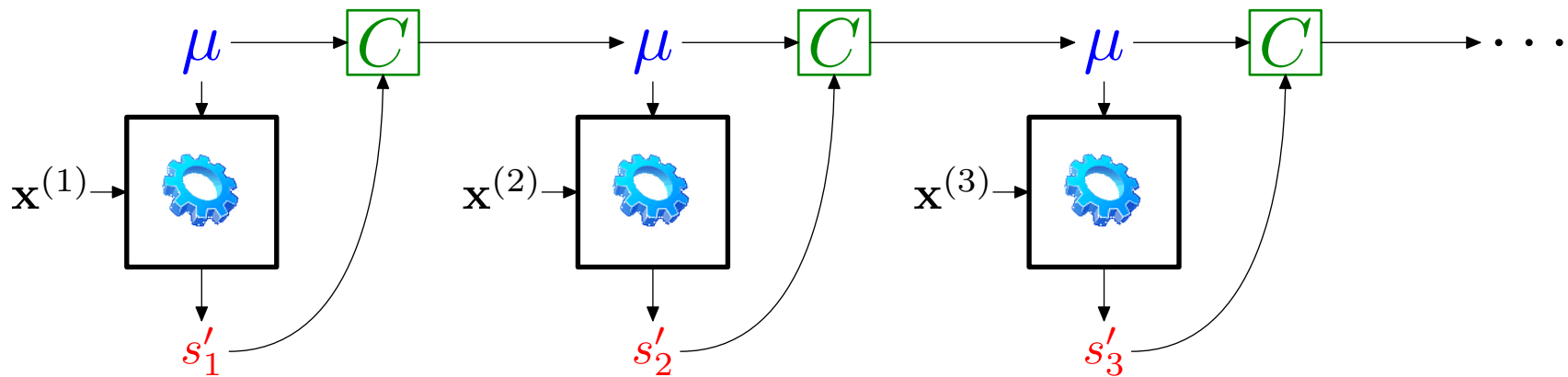
# Online EM [Cappé & Moulines, 2009]



# Online EM [Cappé & Moulines, 2009]

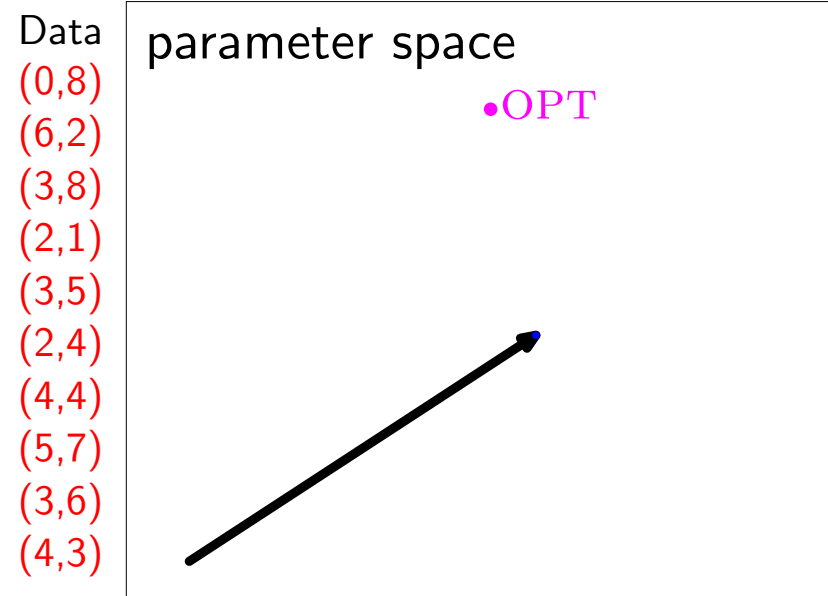


# Online EM [Cappé & Moulines, 2009]



10 data points processed

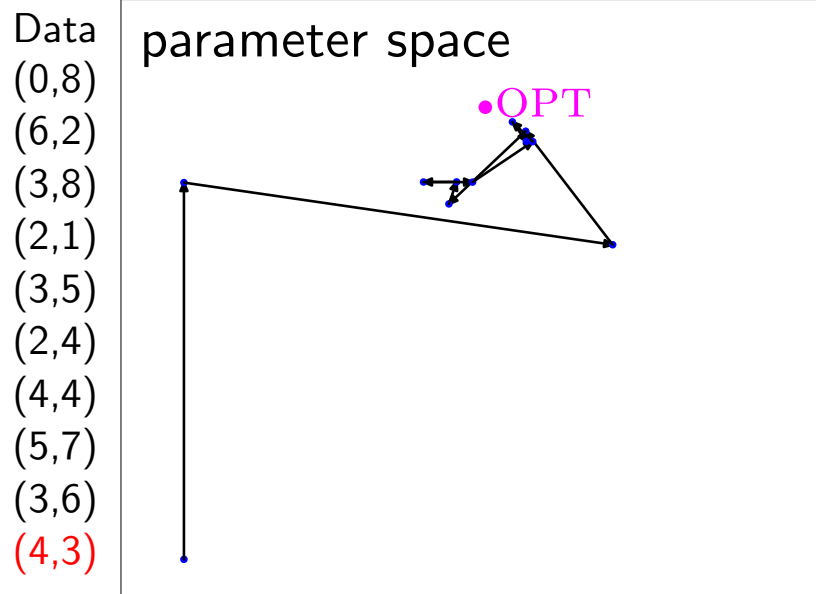
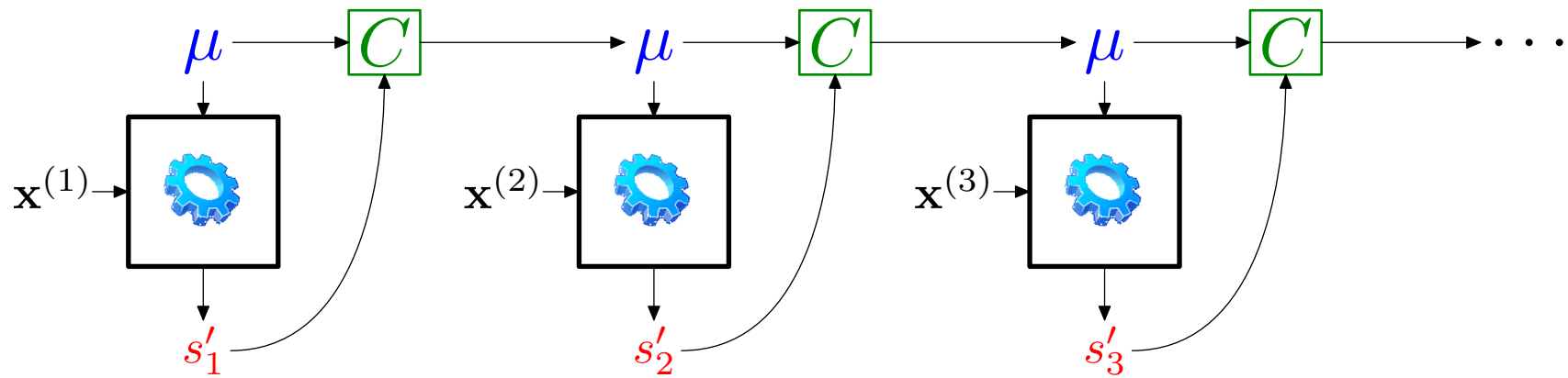
Online (fast, unstable)



10 data points processed

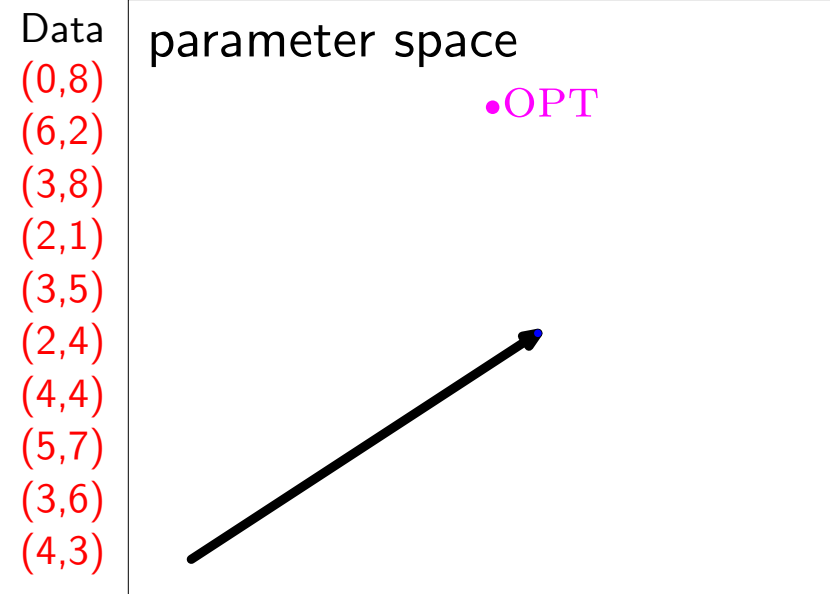
Batch (slow, stable)

# Online EM [Cappé & Moulines, 2009]



10 data points processed

Online (**fast**, **unstable**)

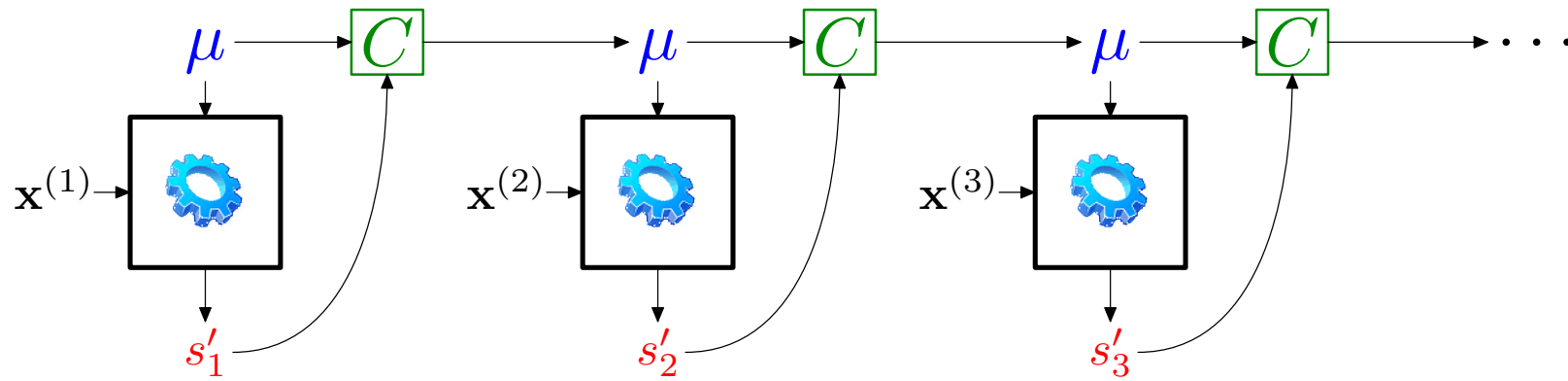


10 data points processed

Batch (**slow**, **stable**)

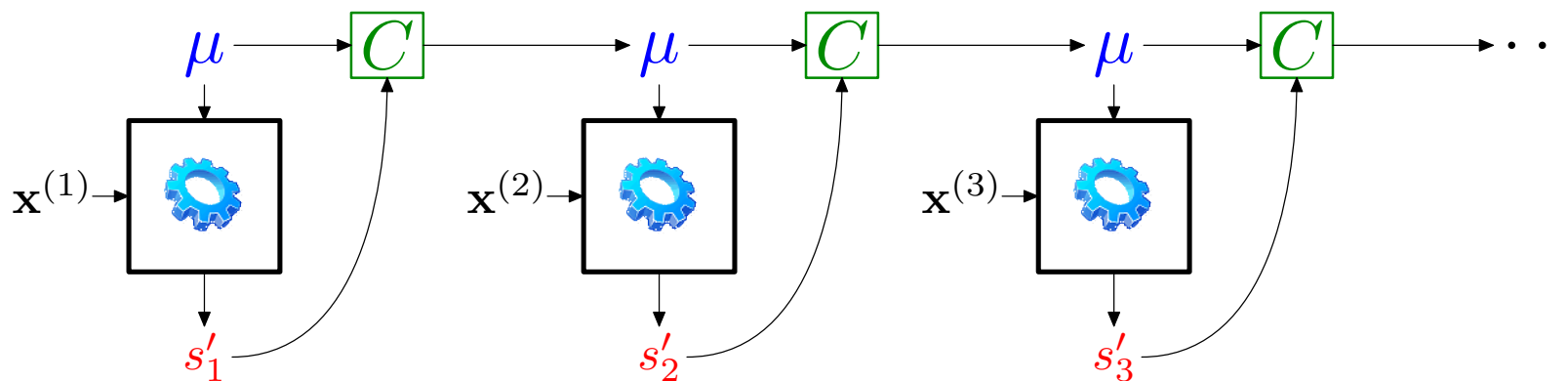
Next: stabilize online EM by modifying optimization parameters

# Optimization parameter 1 of 2: stepsize



Combine old  $\mu$  and new  $s'_i$ :

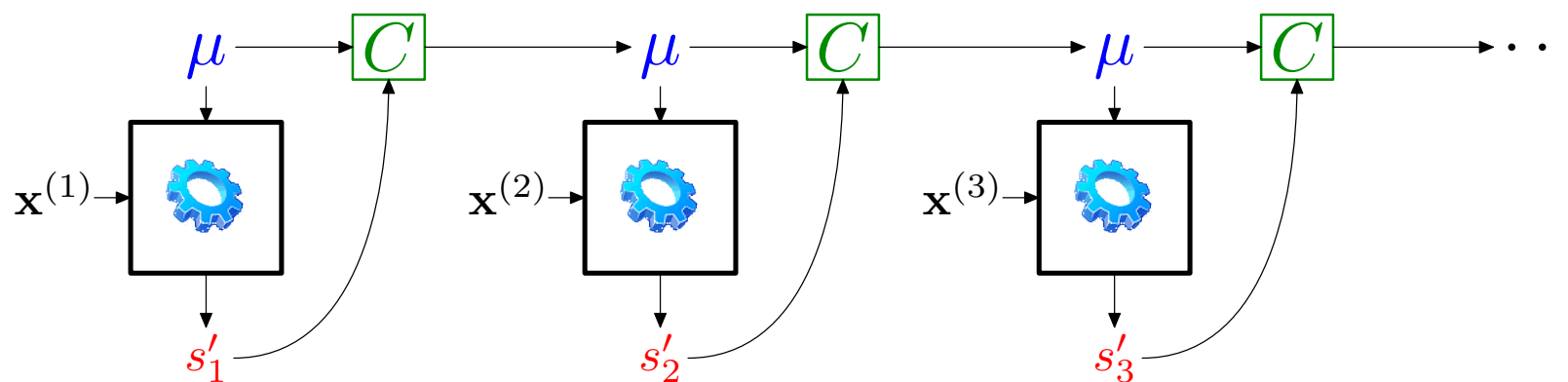
# Optimization parameter 1 of 2: stepsize



Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

# Optimization parameter 1 of 2: stepsize



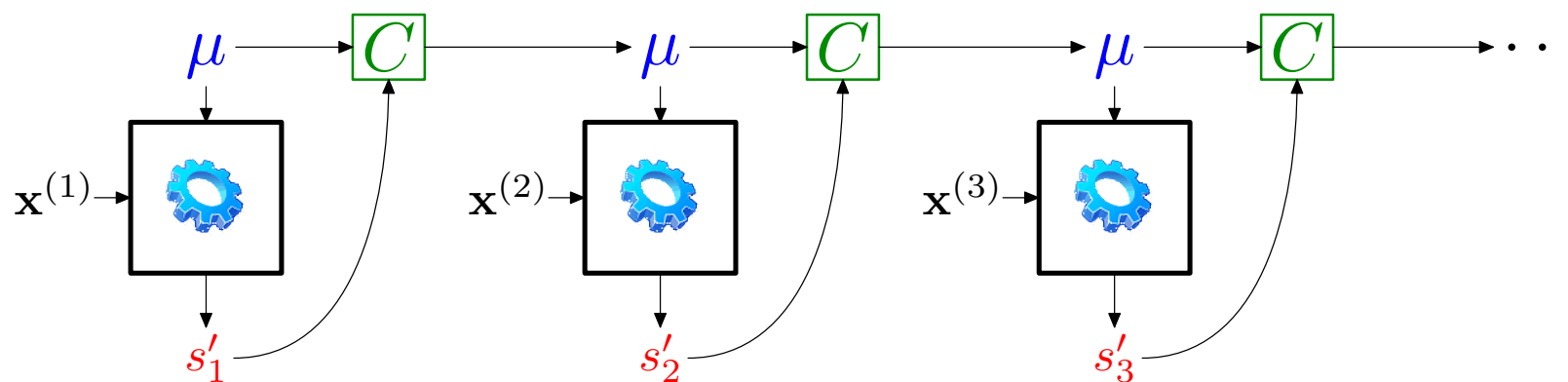
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$$\alpha = \frac{1}{2} \longleftarrow \longrightarrow \alpha = 1$$



# Optimization parameter 1 of 2: stepsize



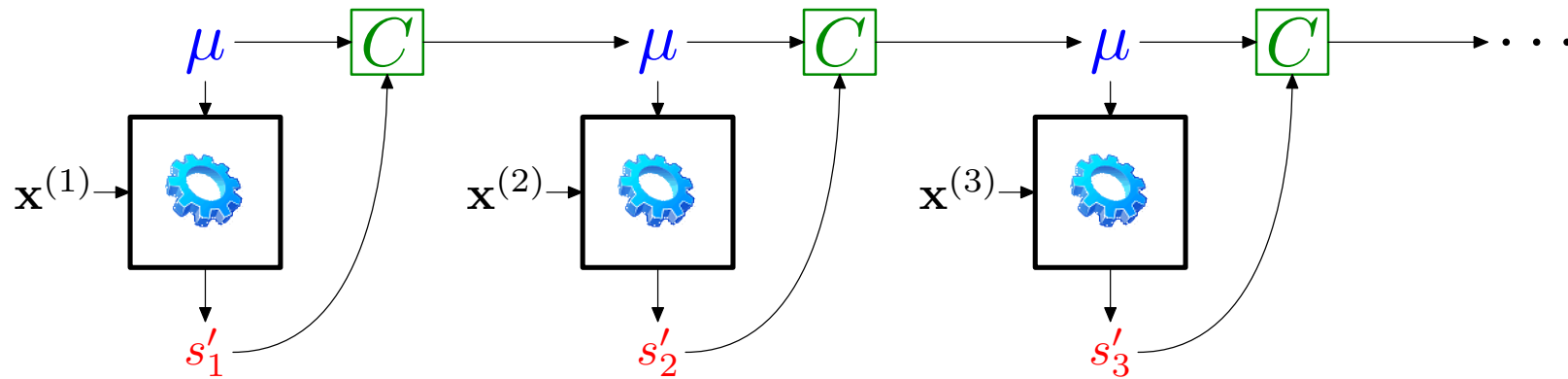
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable

# Optimization parameter 1 of 2: stepsize

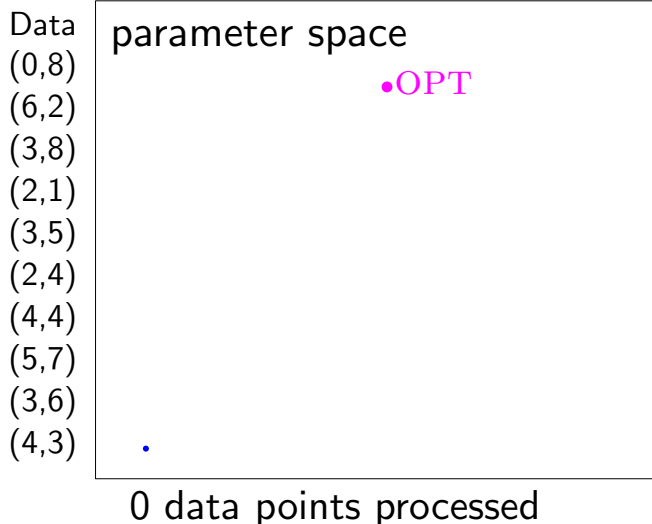


Combine old  $\mu$  and new  $s'_i$ :

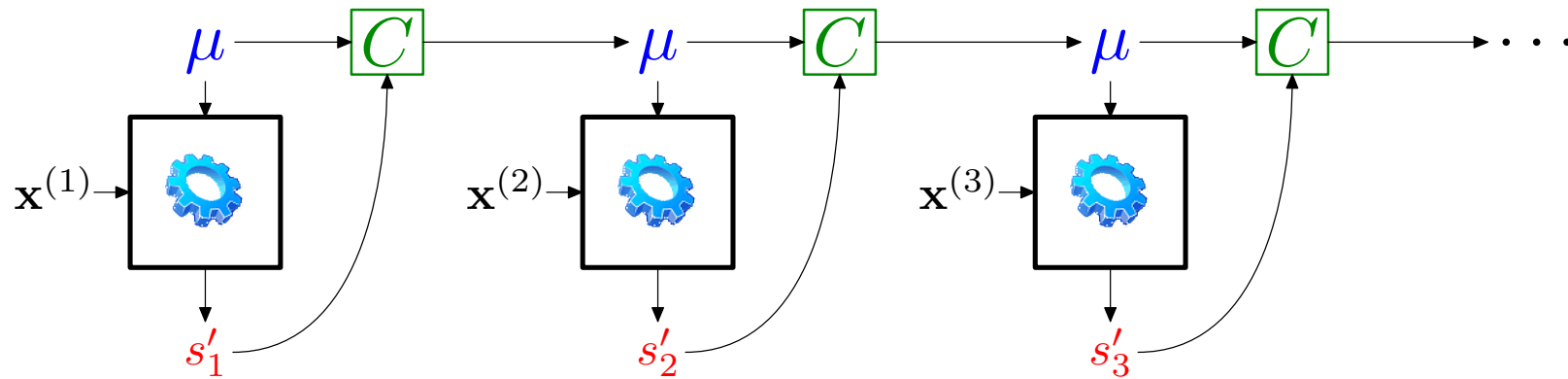
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable



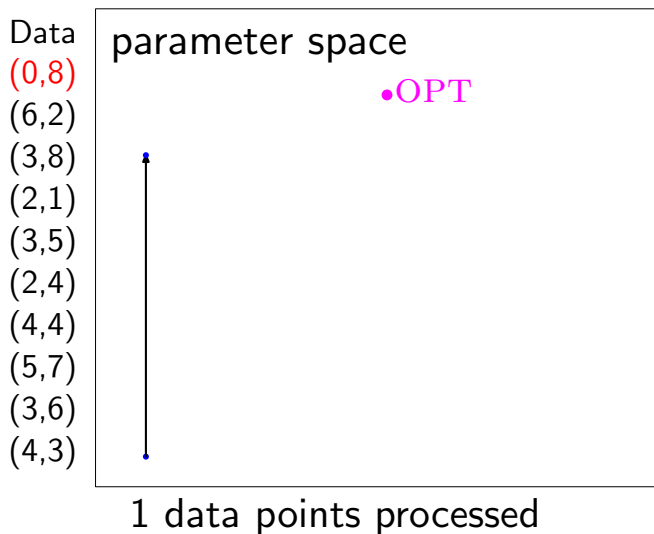
# Optimization parameter 1 of 2: stepsize



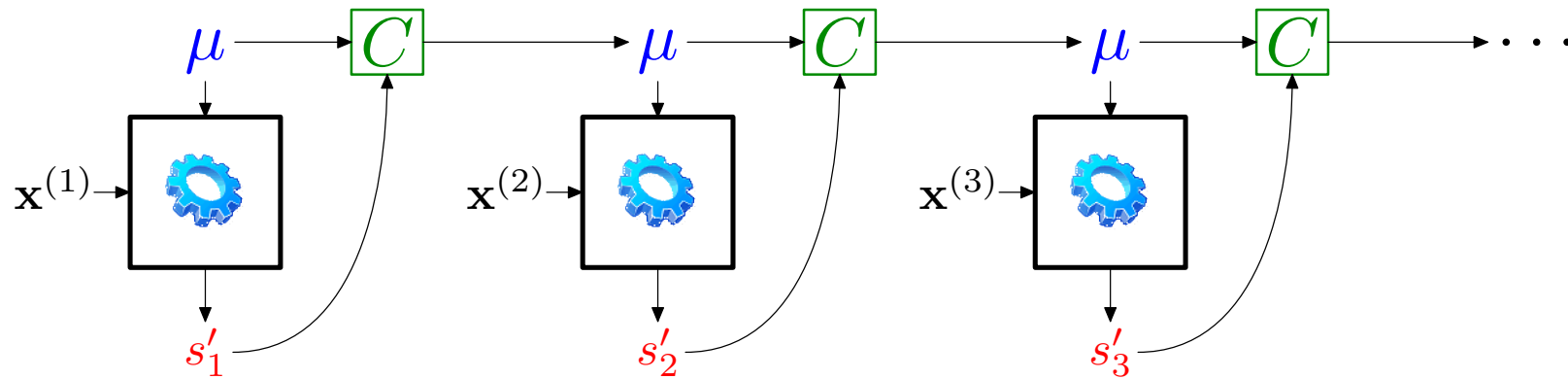
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable
 →  $\alpha = 1$  small updates, stable



# Optimization parameter 1 of 2: stepsize

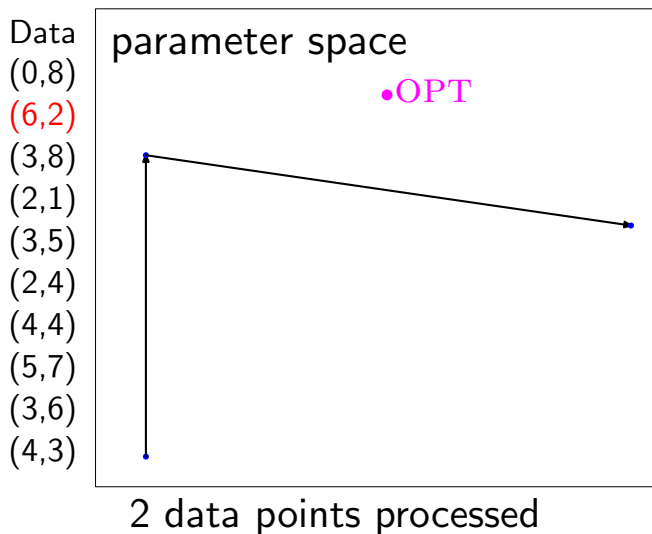


Combine old  $\mu$  and new  $s'_i$ :

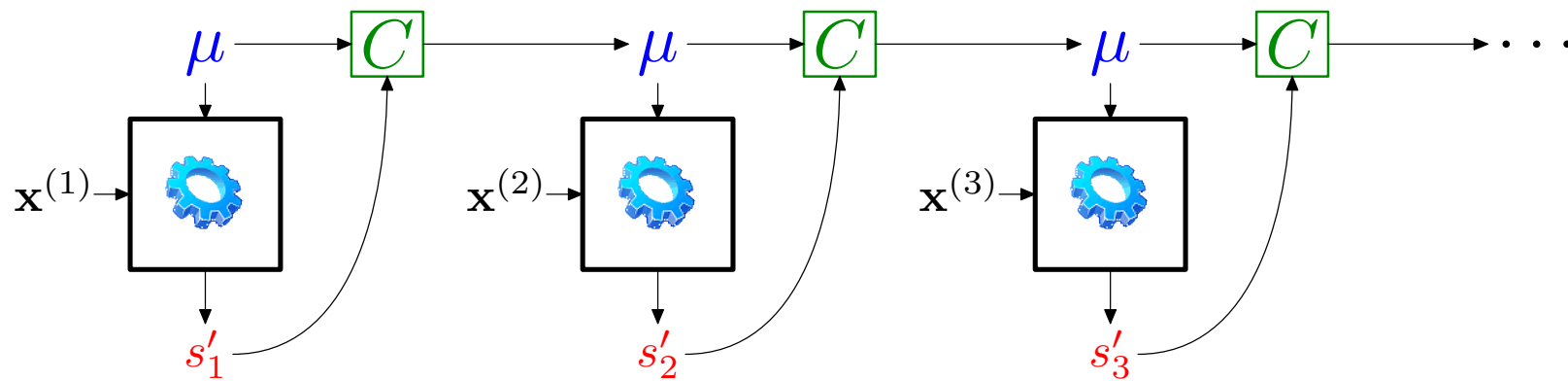
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable



# Optimization parameter 1 of 2: stepsize

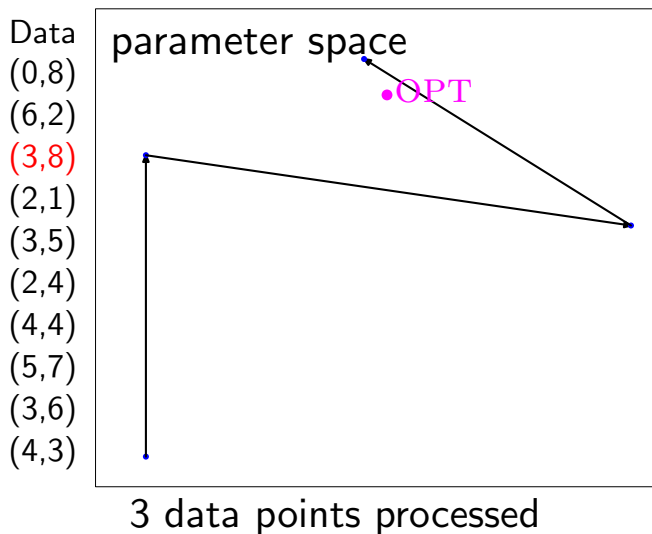


Combine old  $\mu$  and new  $s'_i$ :

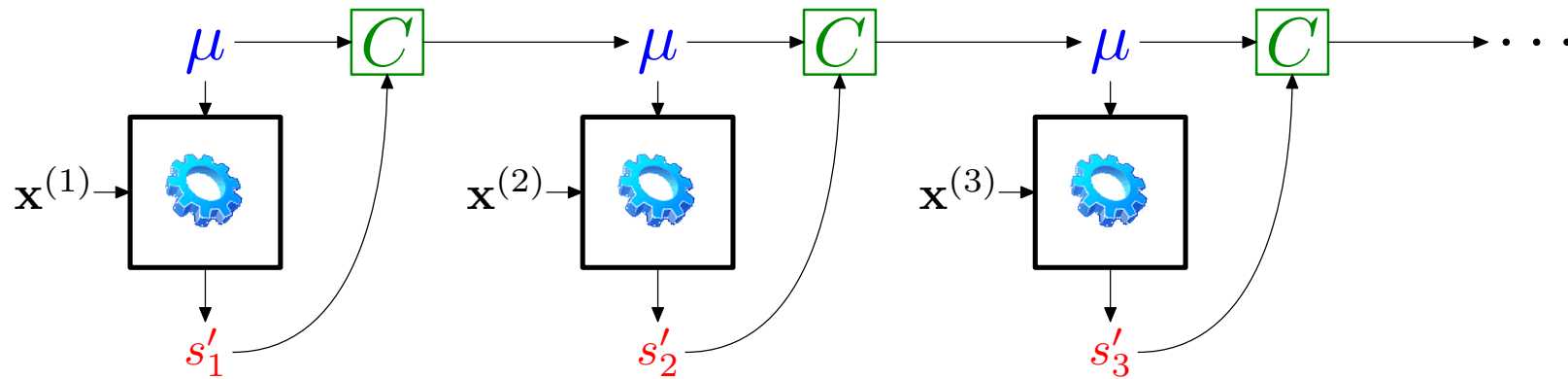
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable



# Optimization parameter 1 of 2: stepsize

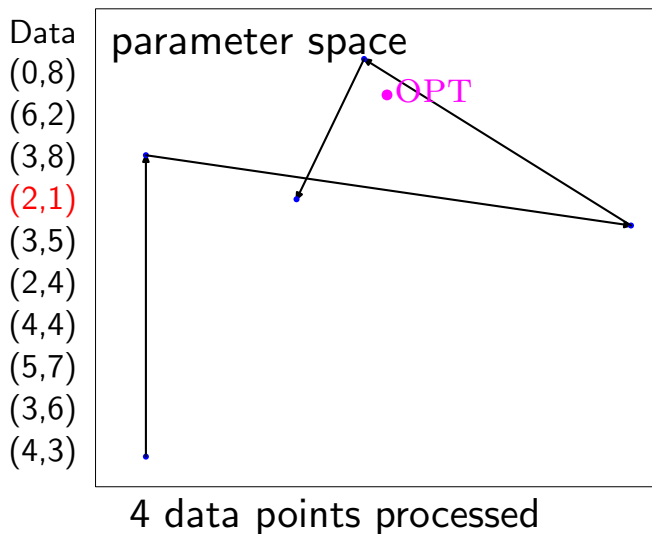


Combine old  $\mu$  and new  $s'_i$ :

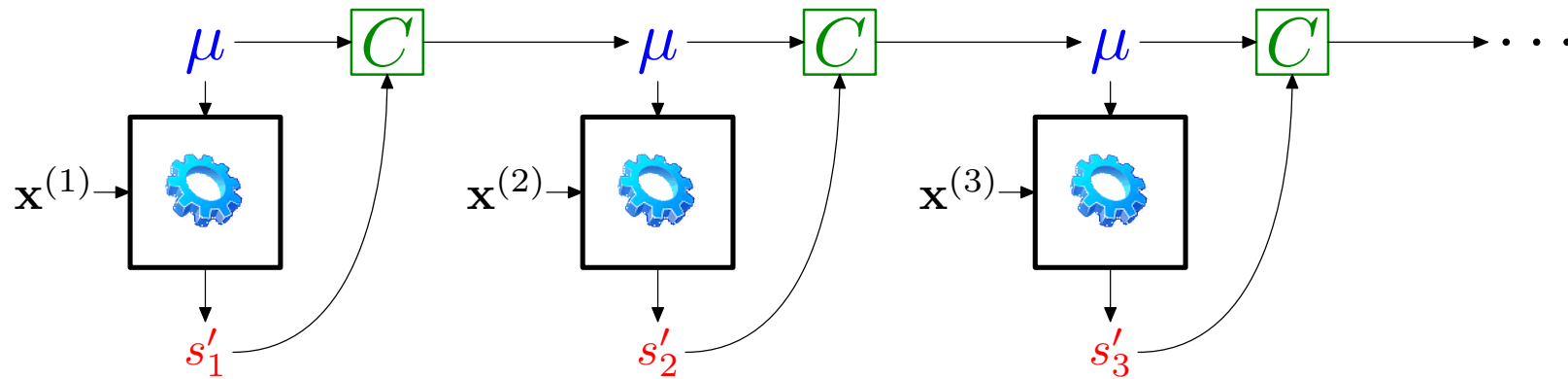
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable



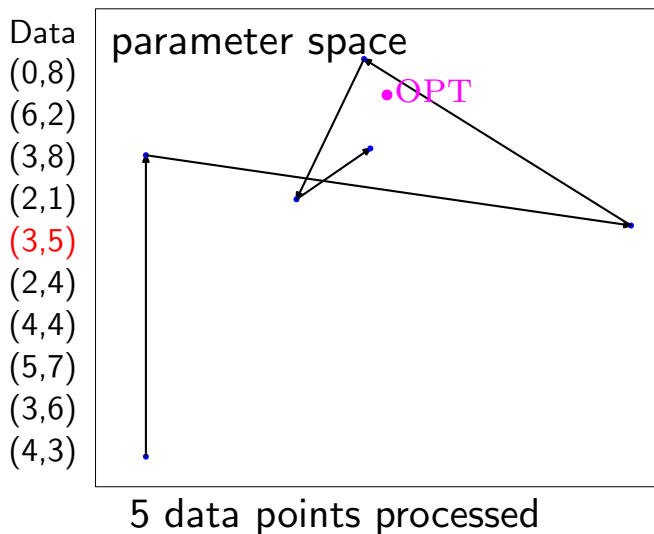
# Optimization parameter 1 of 2: stepsize



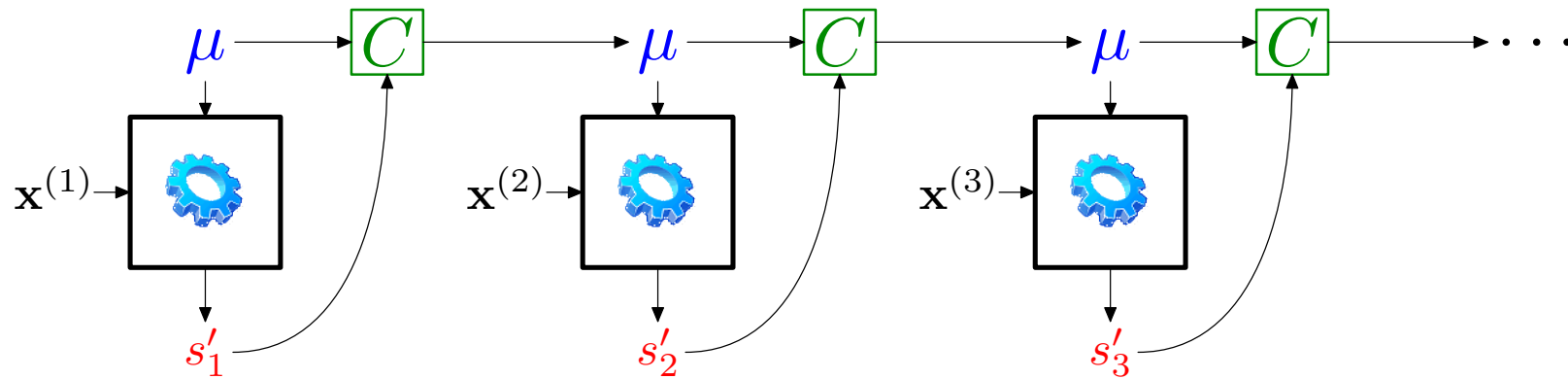
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable
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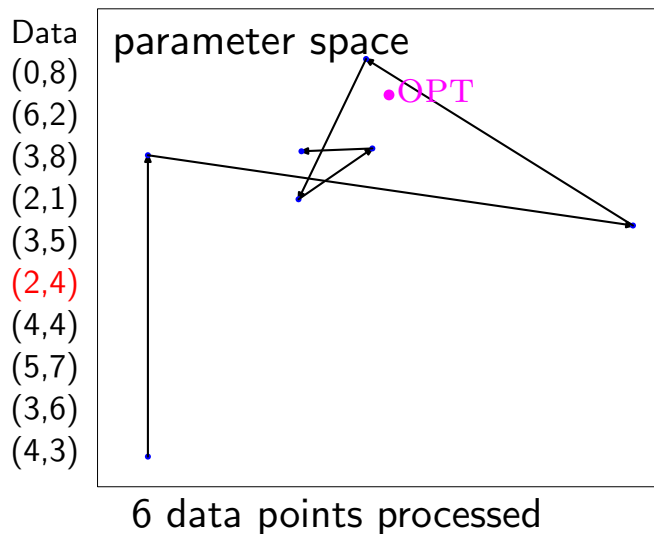
# Optimization parameter 1 of 2: stepsize



Combine old  $\mu$  and new  $s'_i$ :

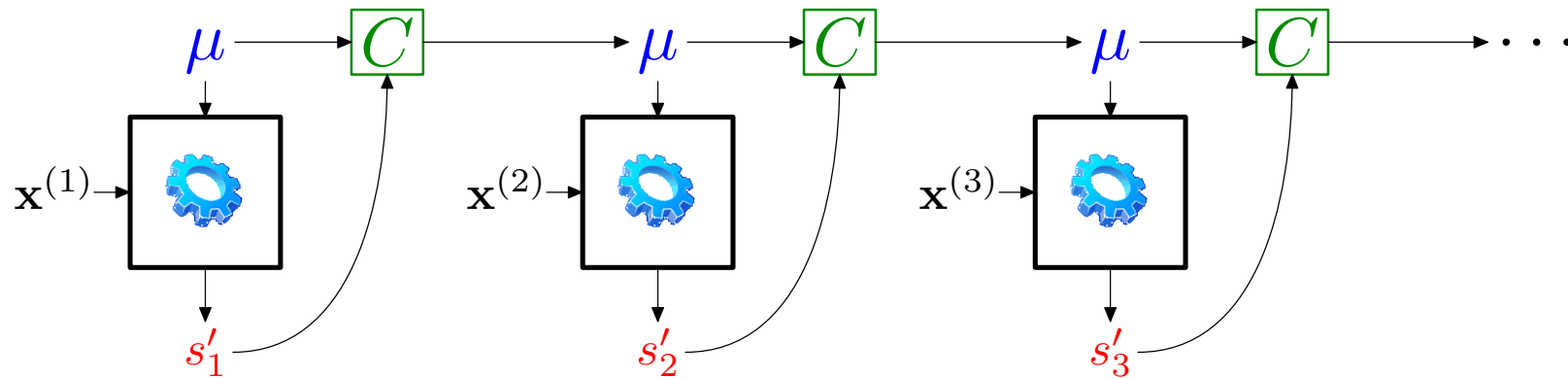
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable →  $\alpha = 1$  small updates, stable





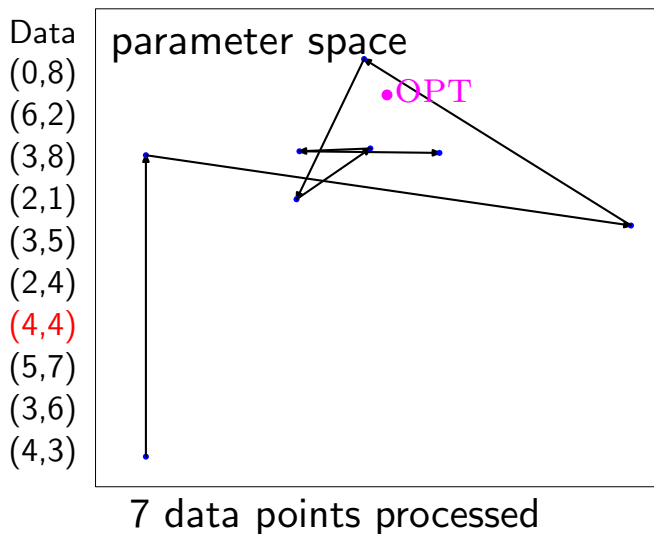
# Optimization parameter 1 of 2: stepsize



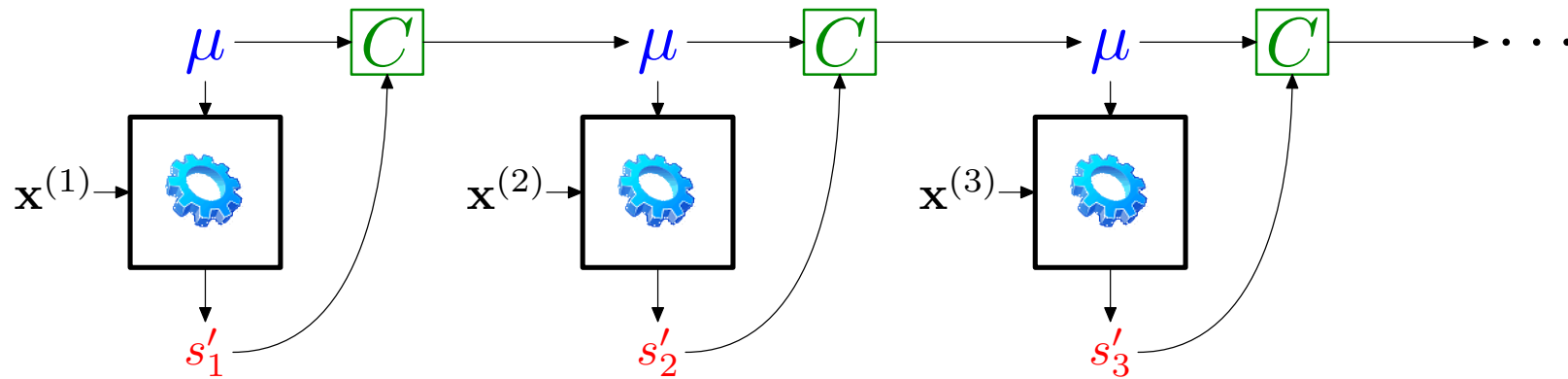
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable
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# Optimization parameter 1 of 2: stepsize

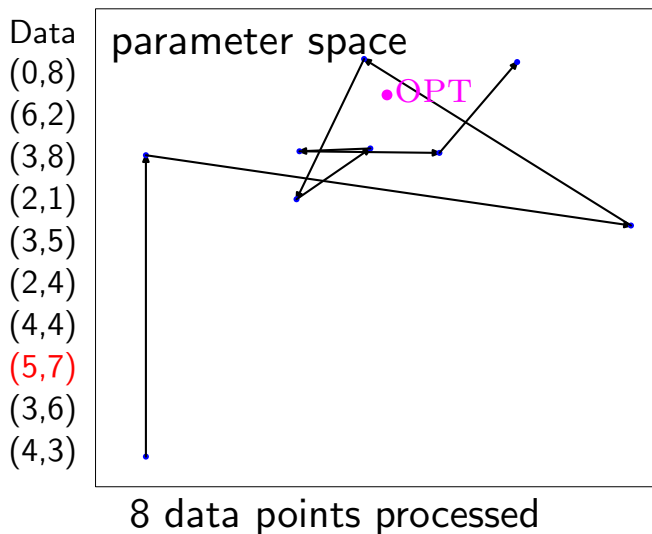


Combine old  $\mu$  and new  $s'_i$ :

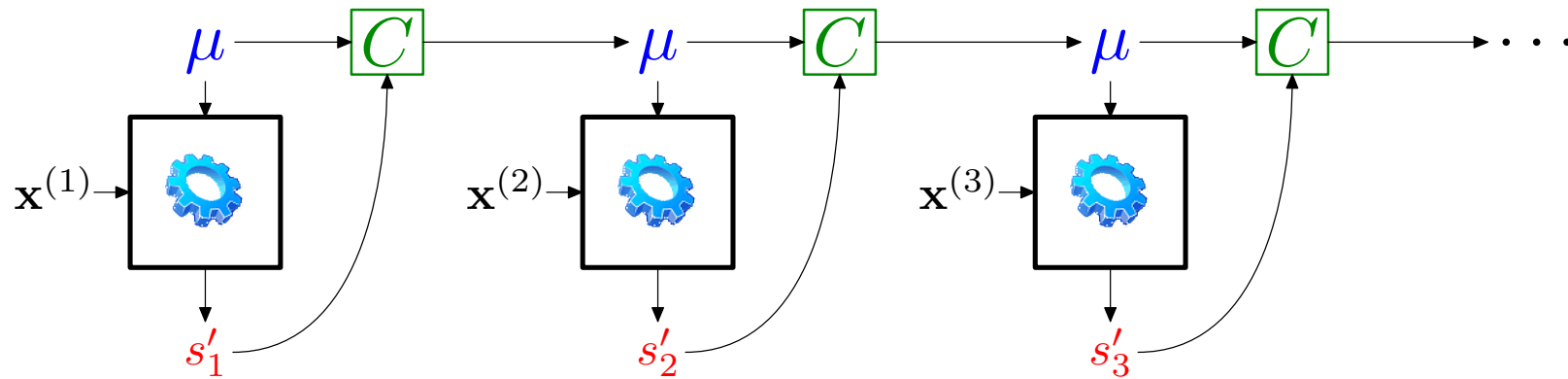
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

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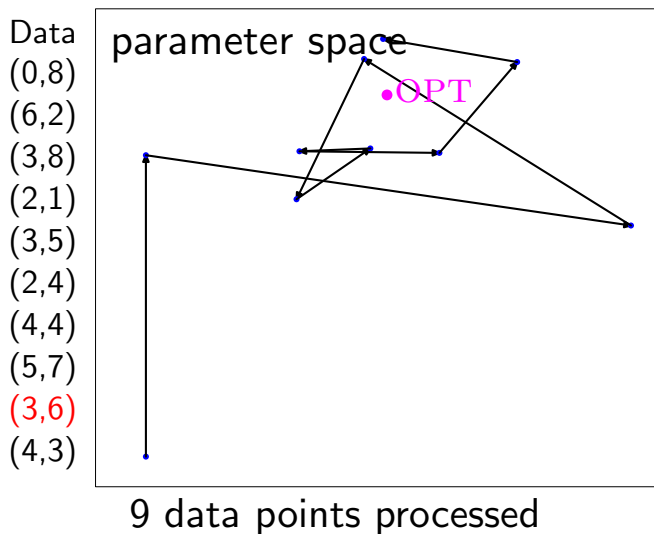
# Optimization parameter 1 of 2: stepsize



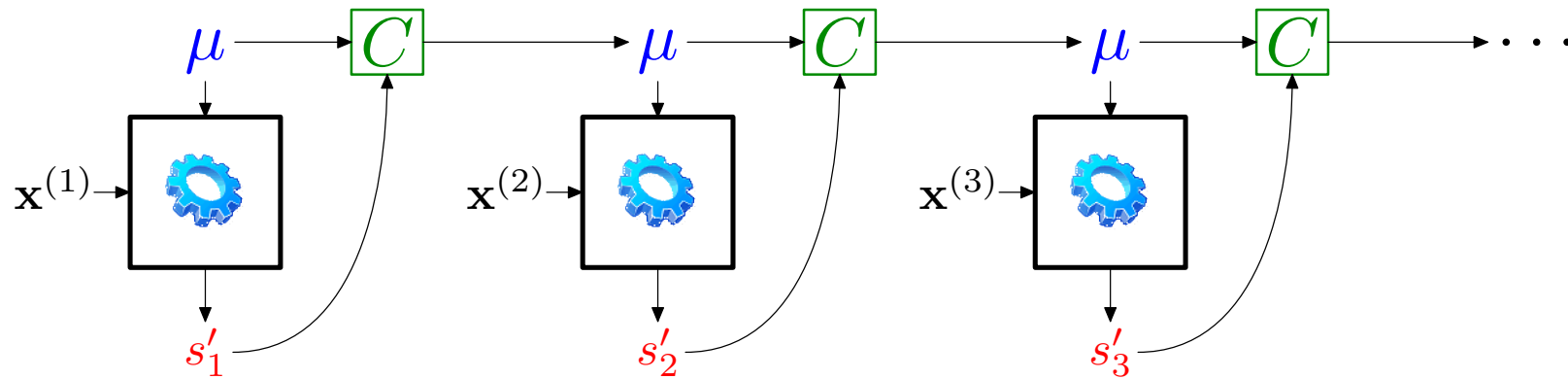
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable
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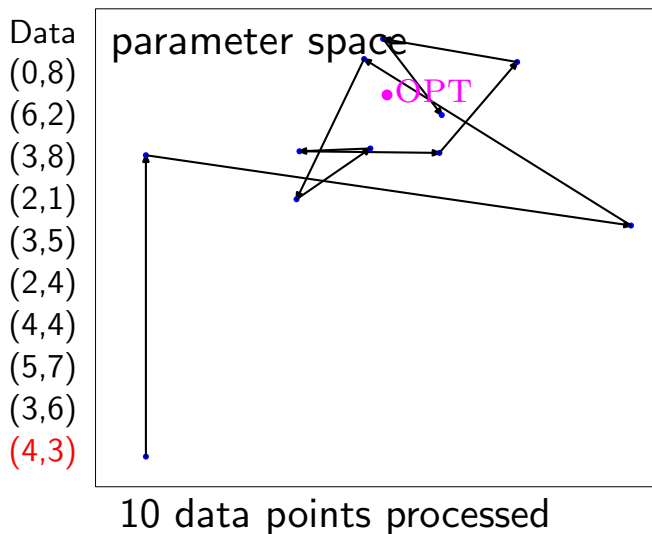
# Optimization parameter 1 of 2: stepsize



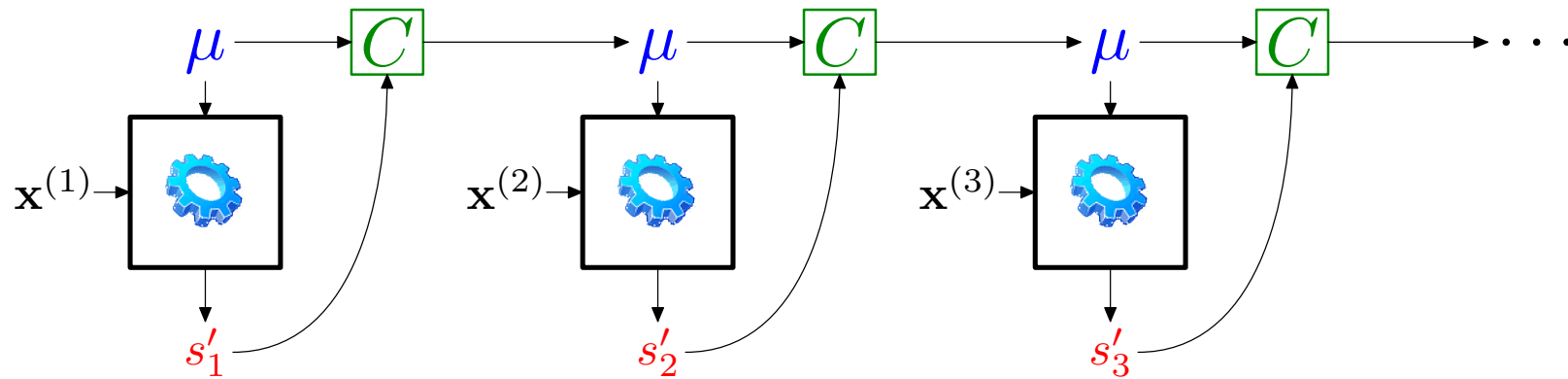
Combine old  $\mu$  and new  $s'_i$ :

$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

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# Optimization parameter 1 of 2: stepsize

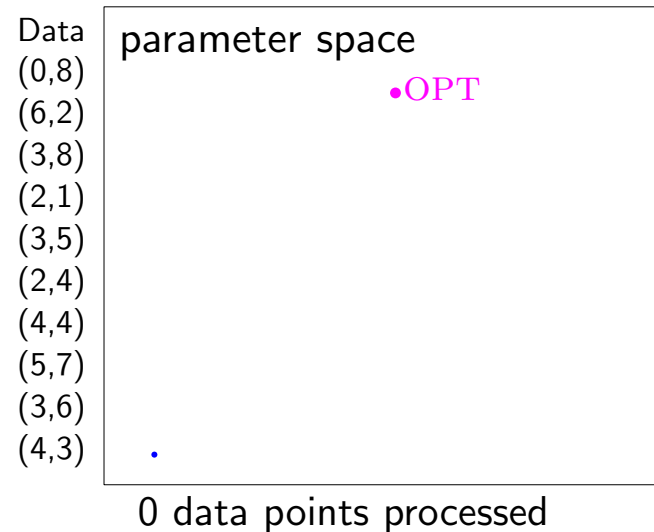
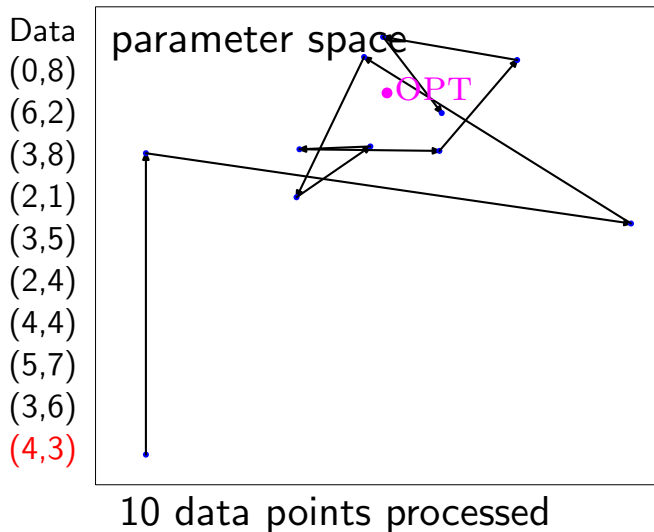


Combine old  $\mu$  and new  $s'_i$ :

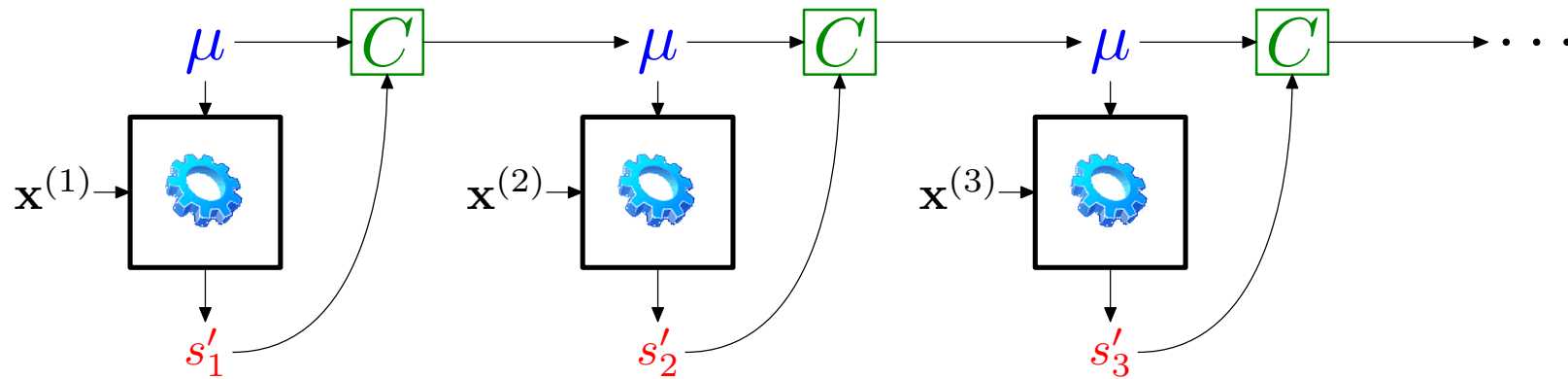
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$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable



# Optimization parameter 1 of 2: stepsize

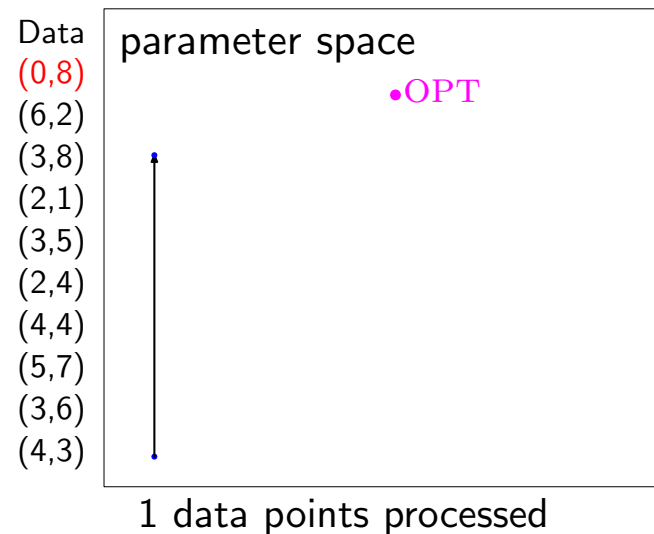
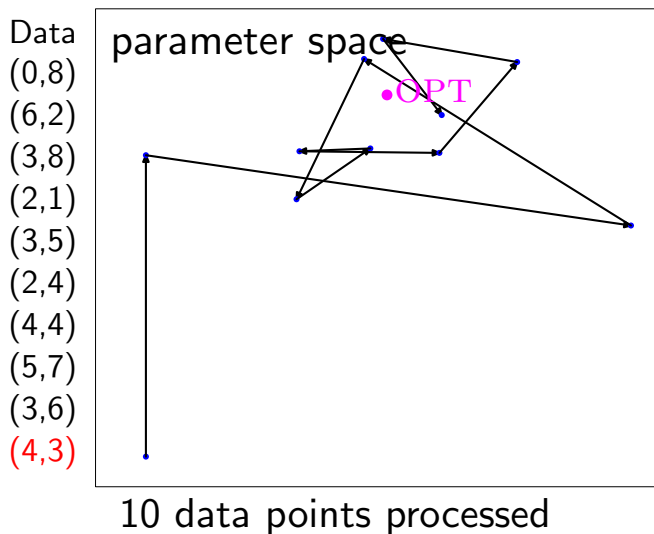


Combine old  $\mu$  and new  $s'_i$ :

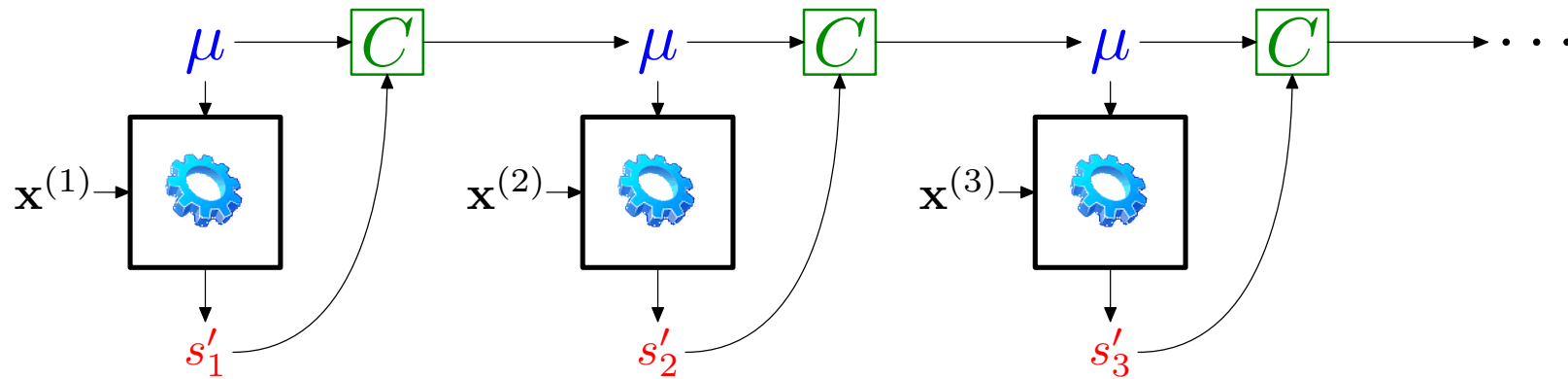
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# Optimization parameter 1 of 2: stepsize

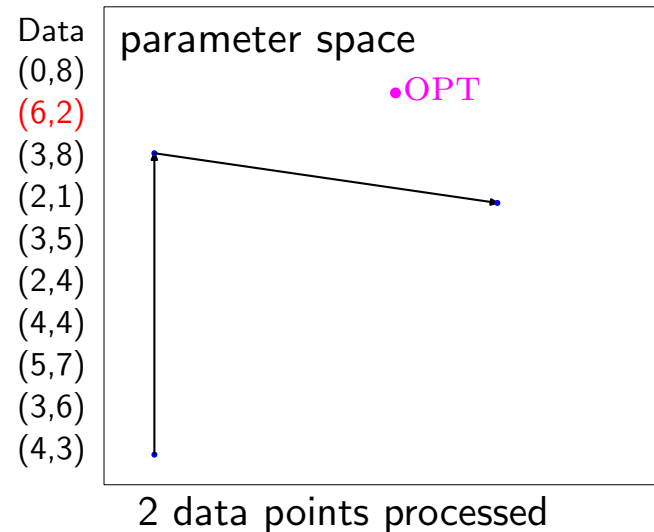
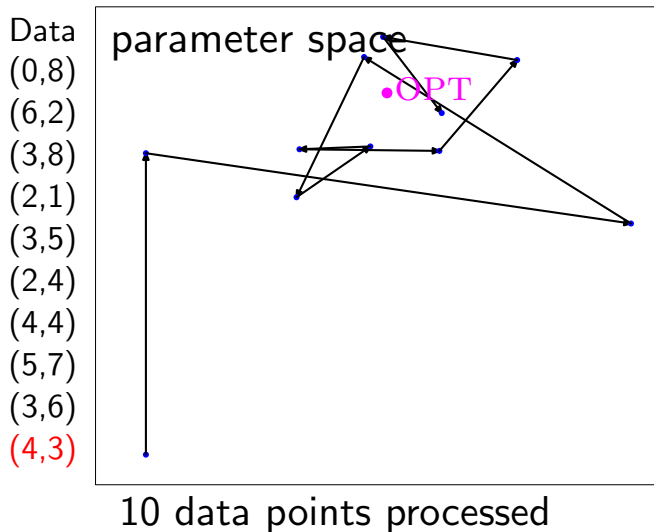


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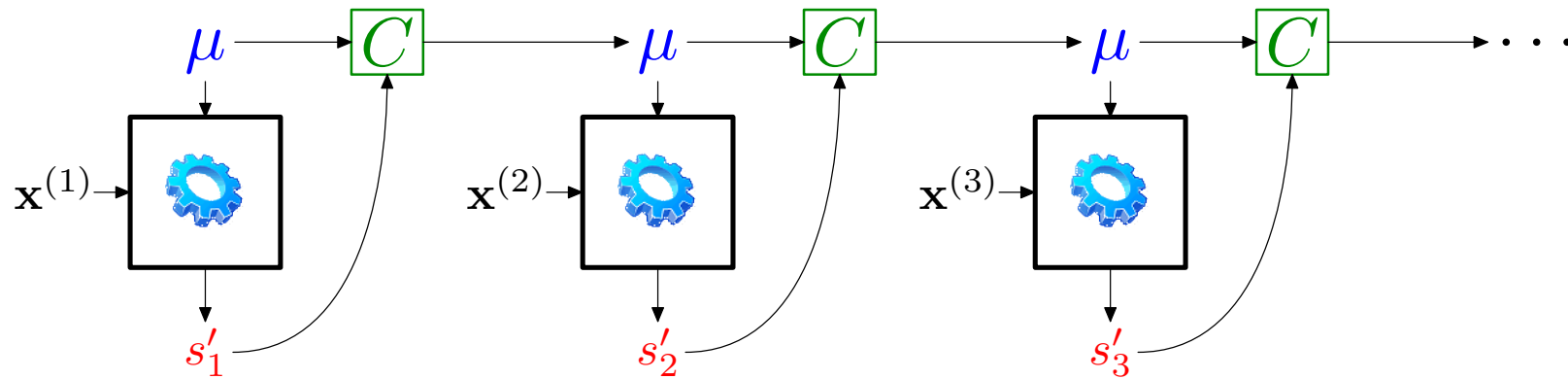
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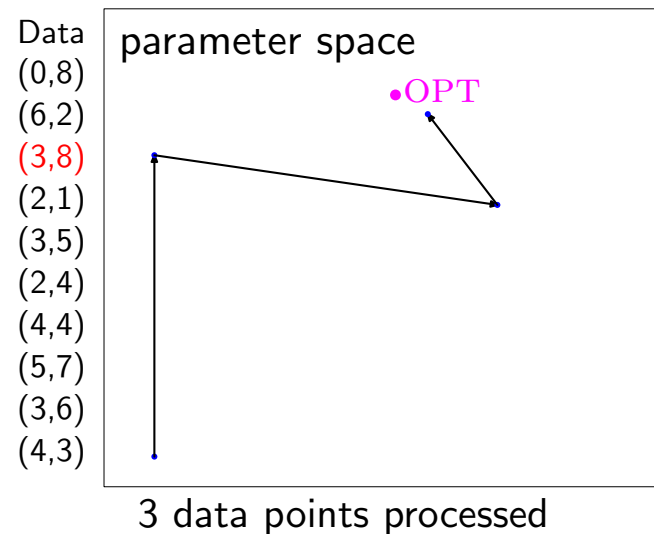
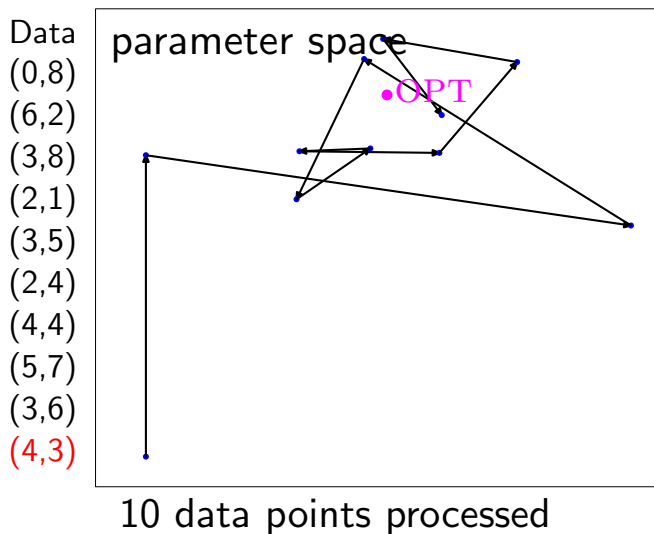
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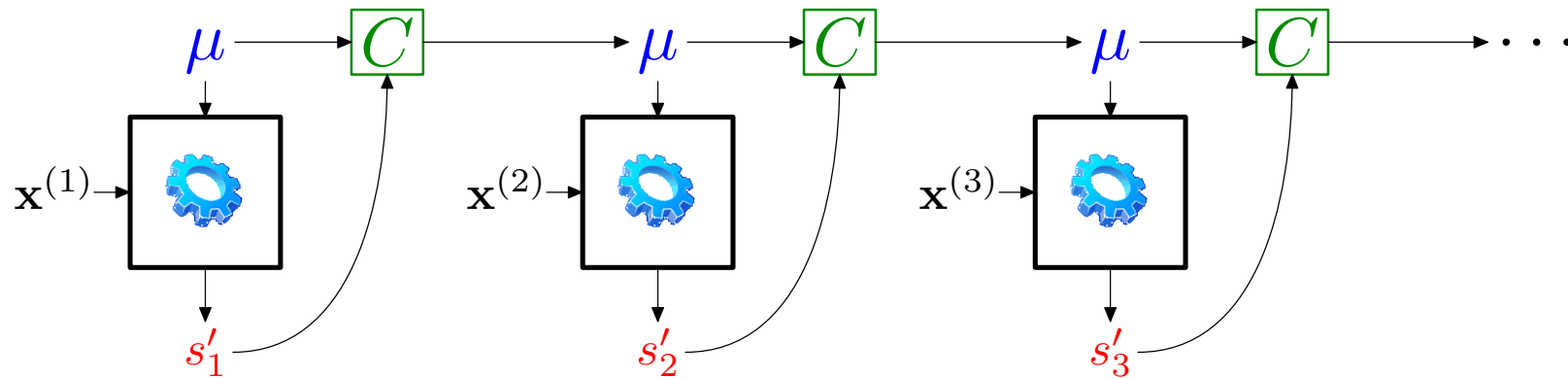
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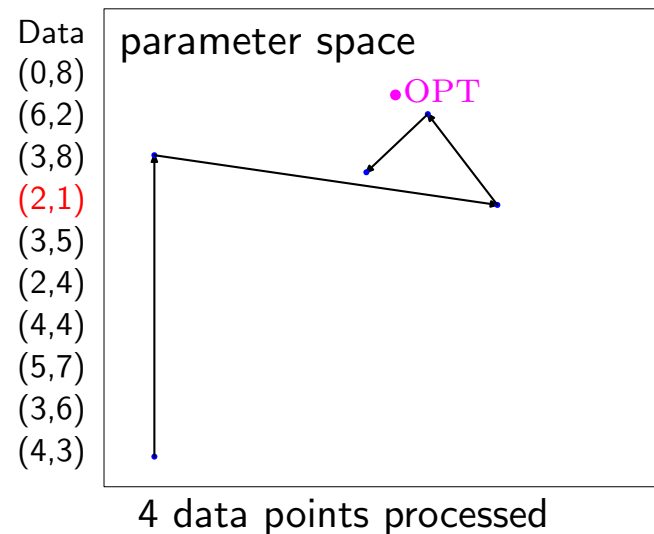
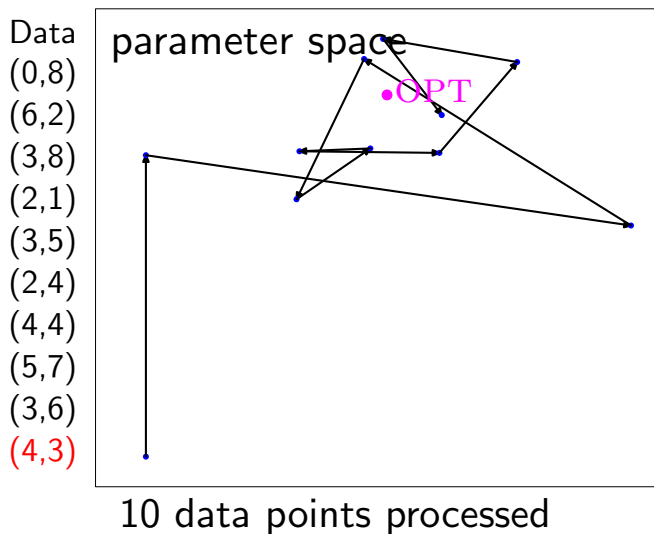


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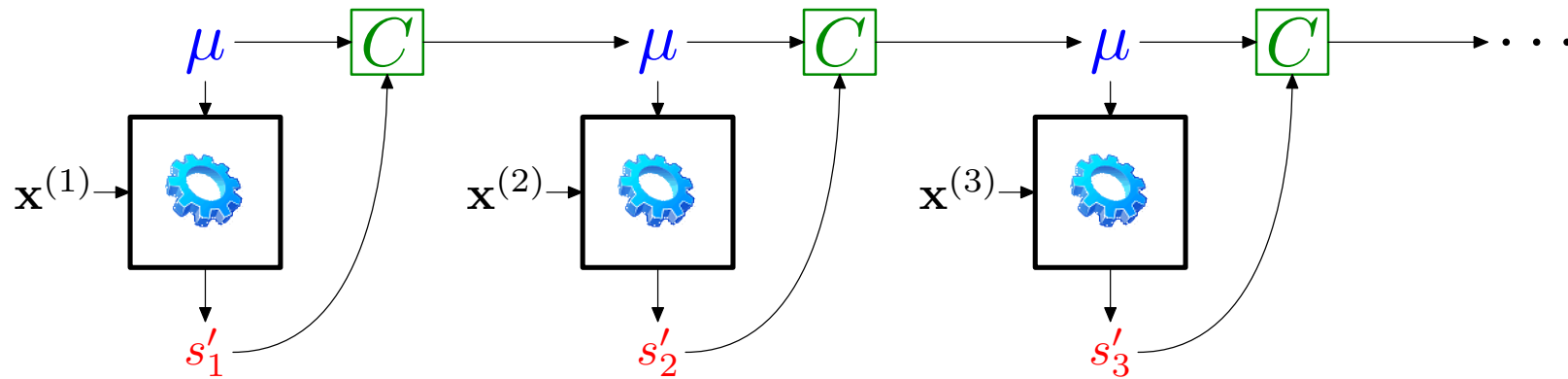
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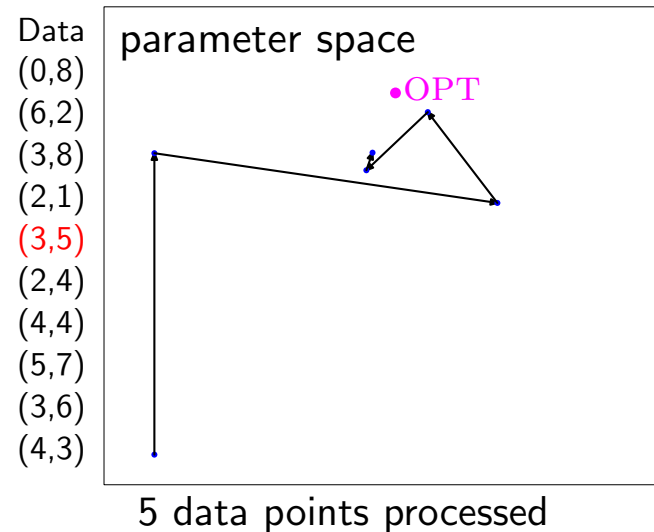
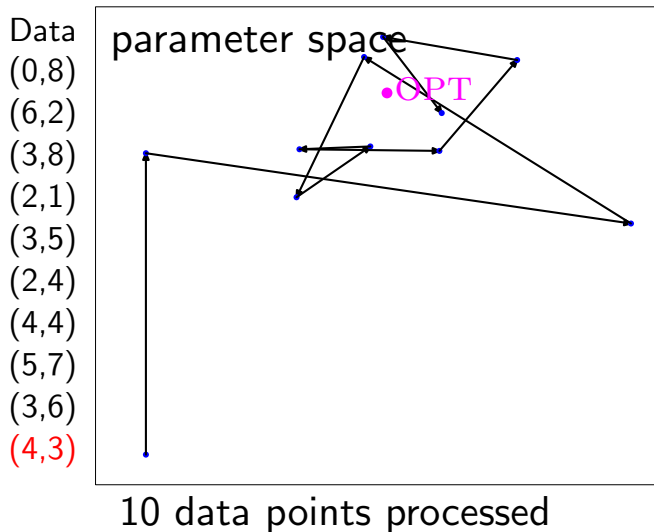


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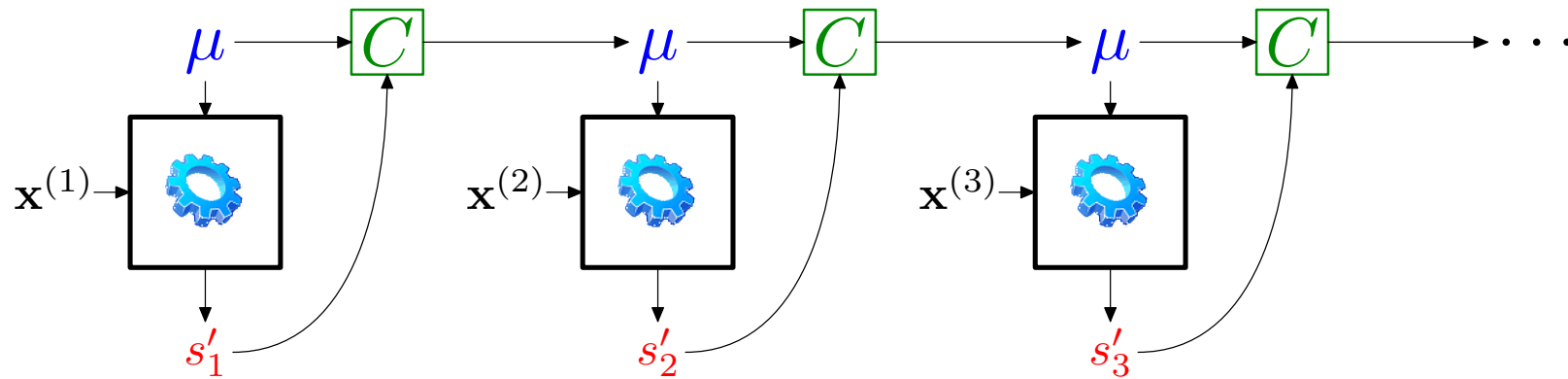
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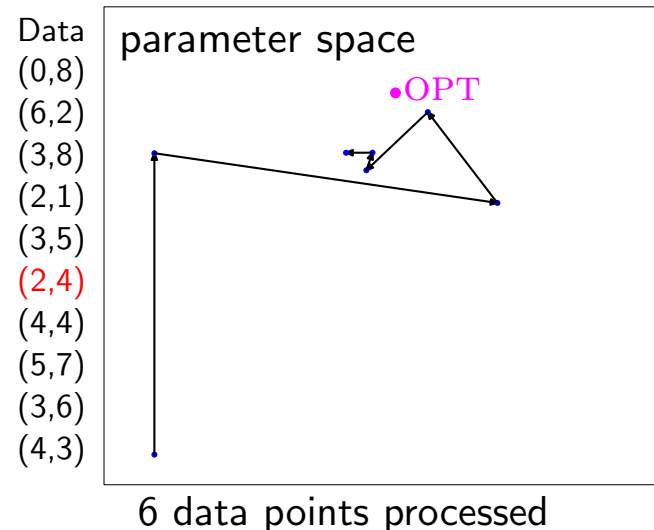
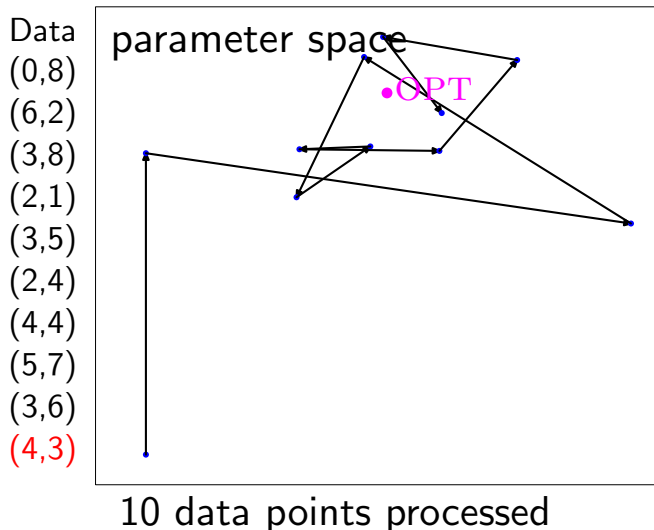


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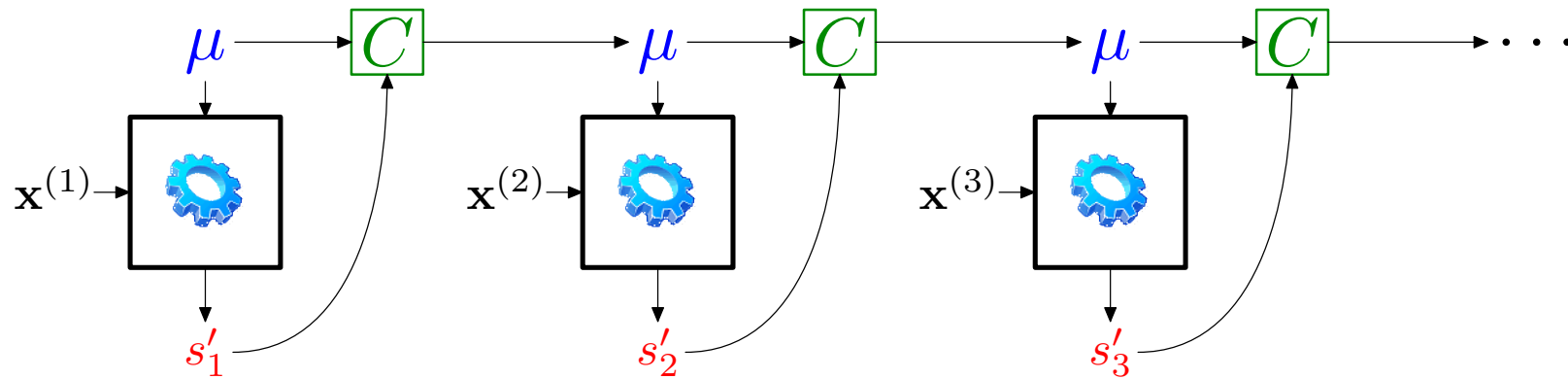
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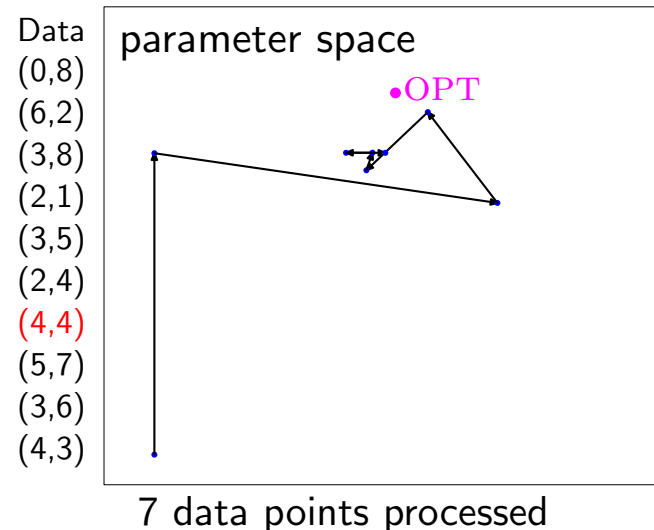
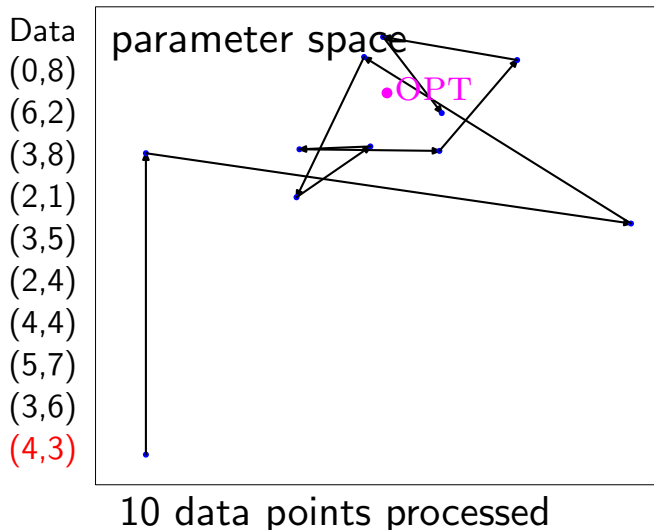


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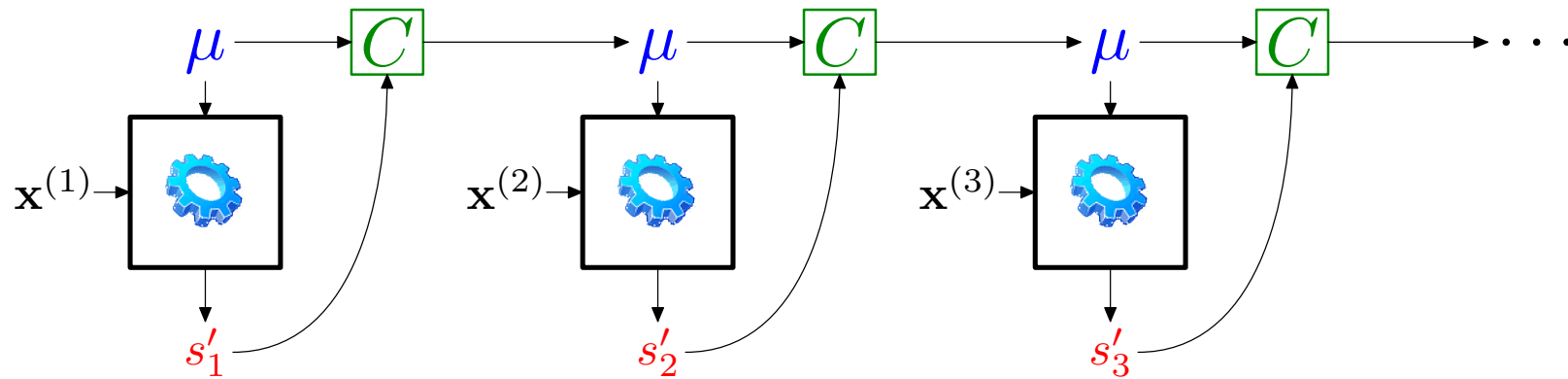
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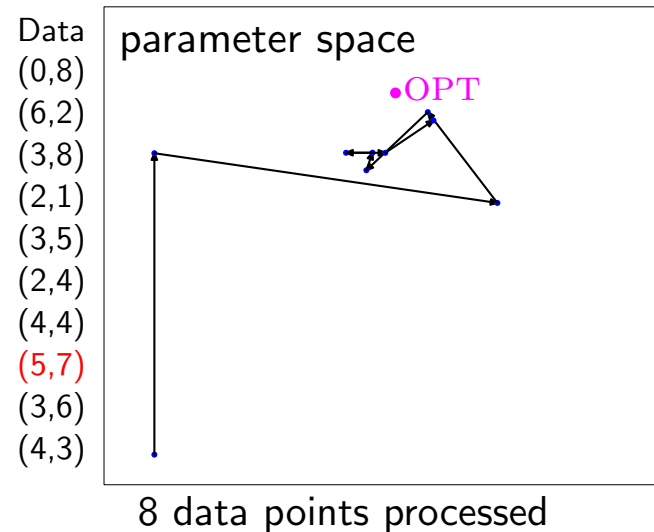
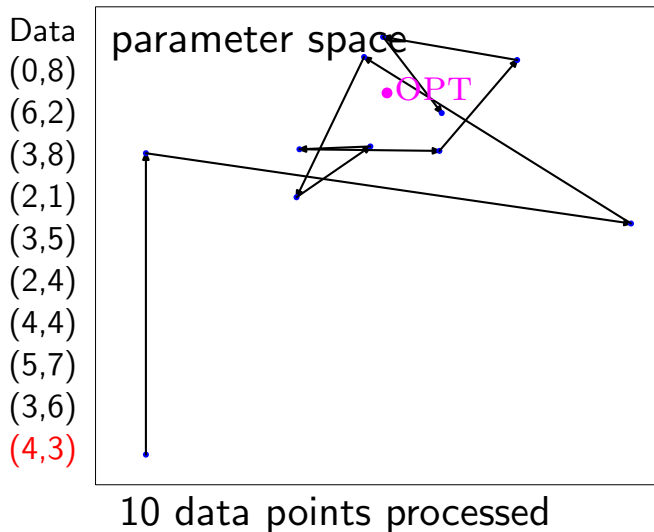


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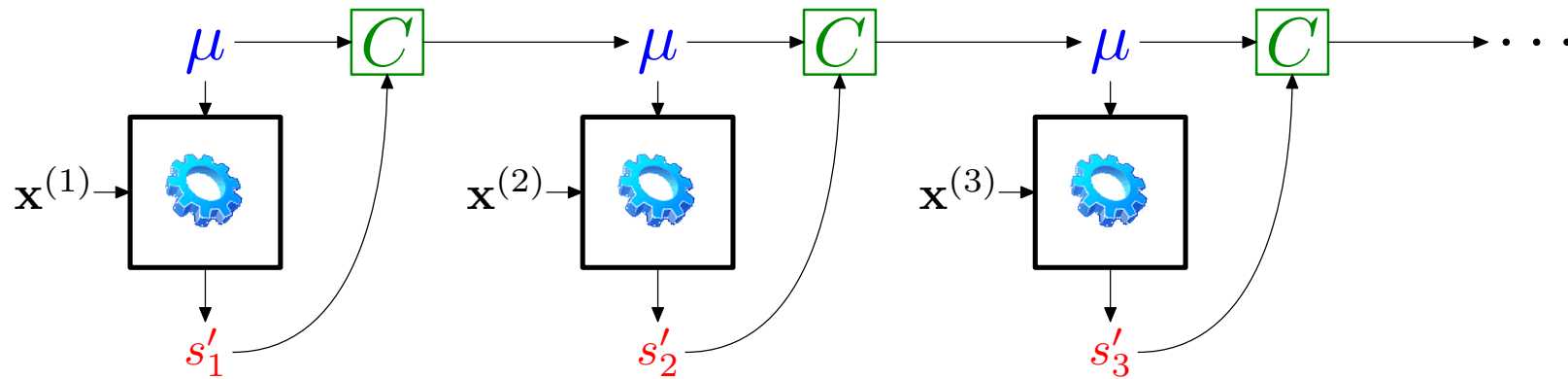
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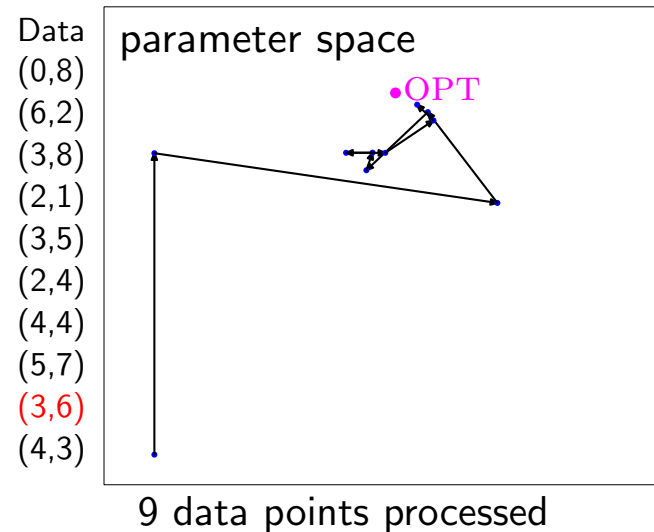
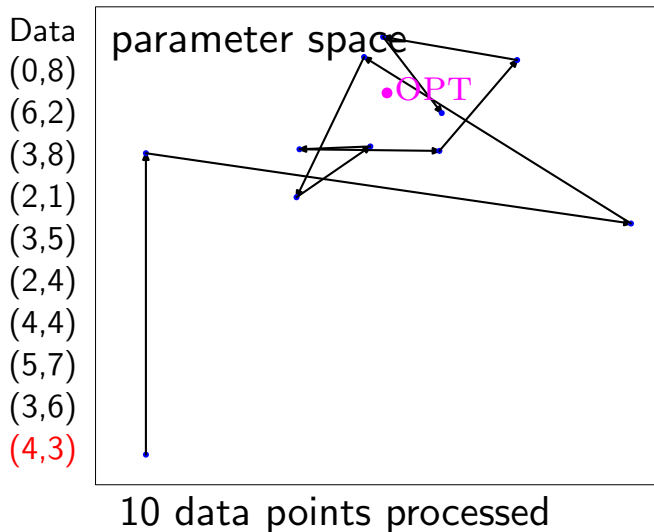


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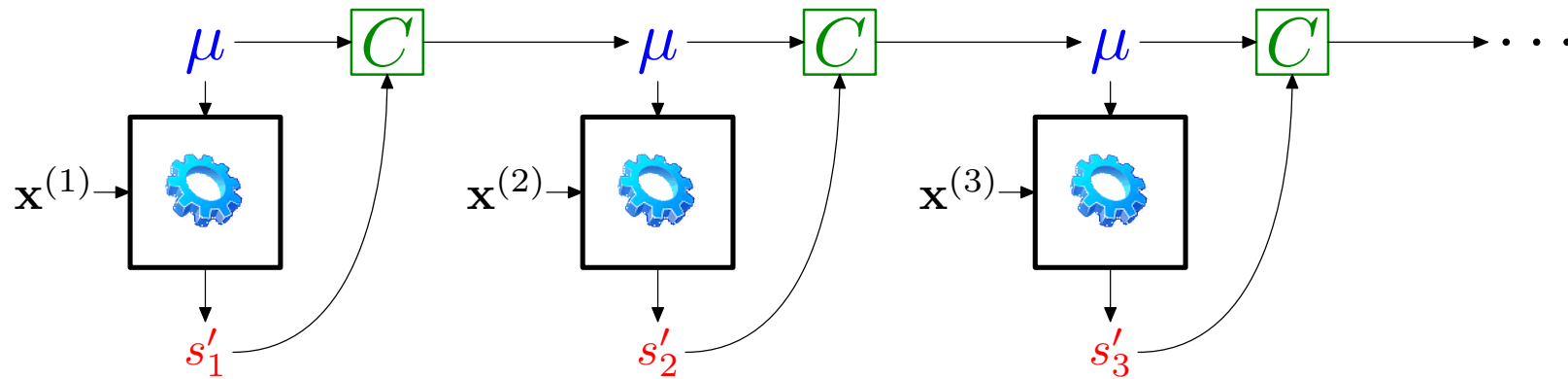
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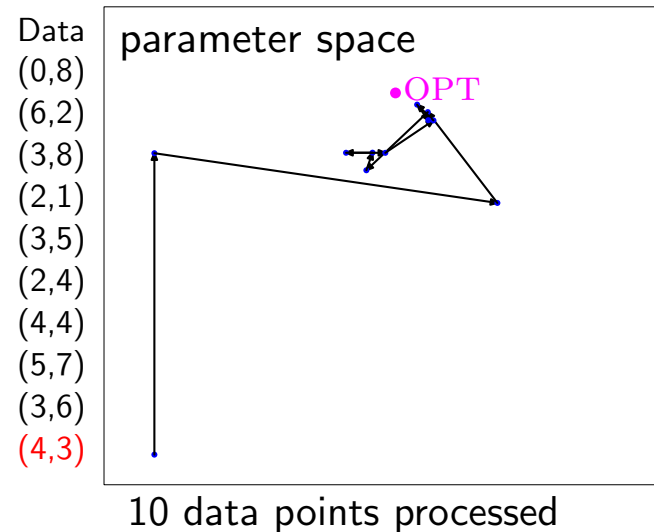
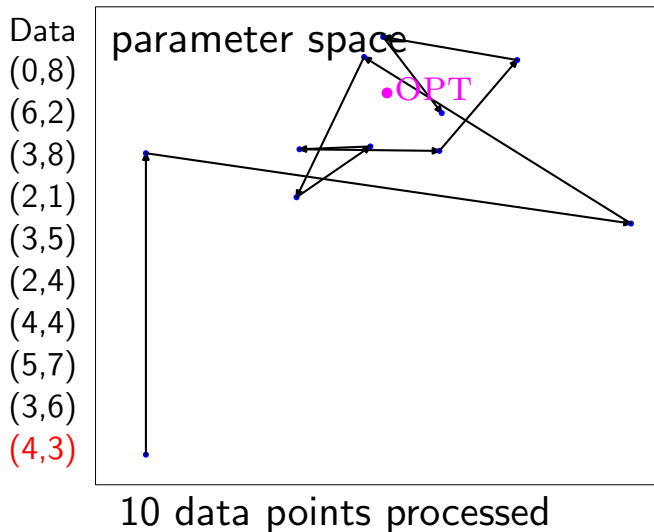
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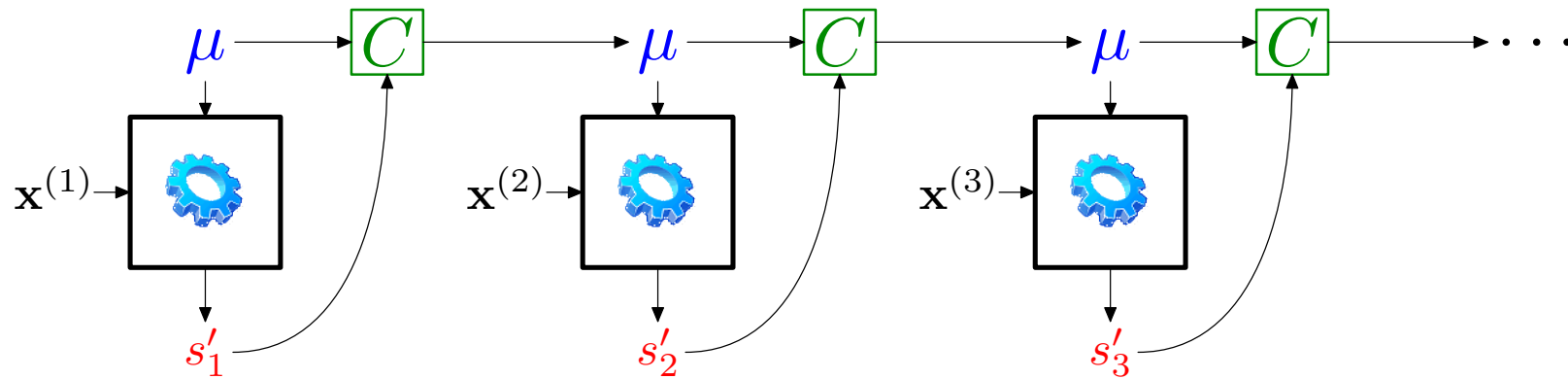
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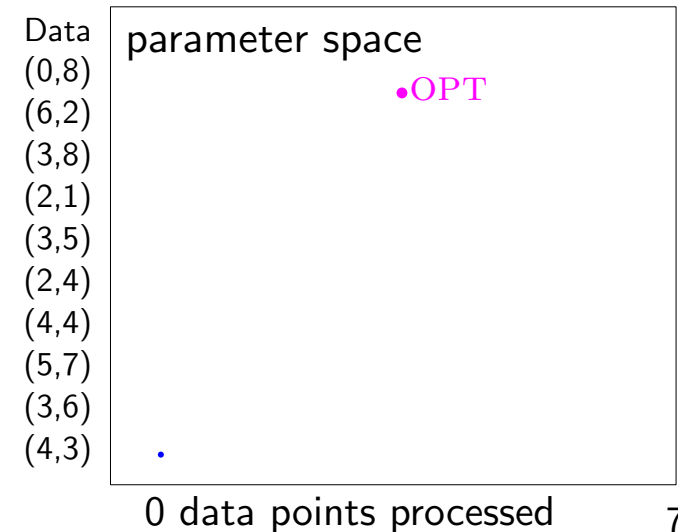
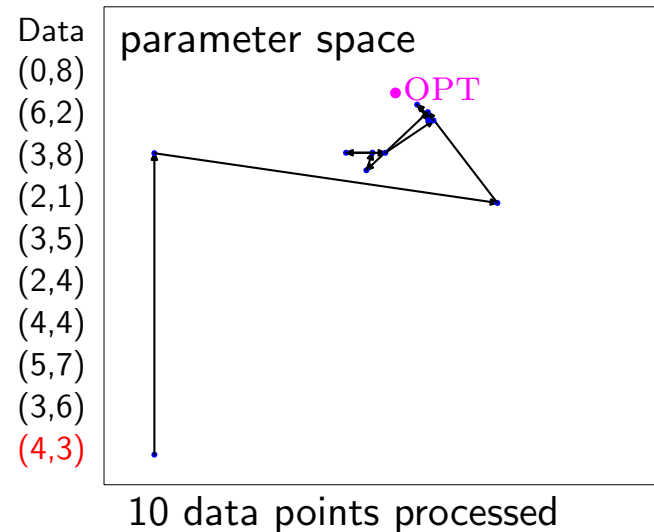
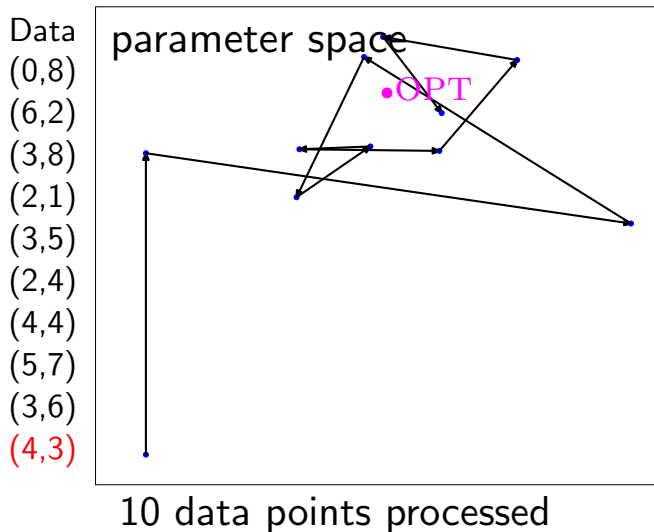


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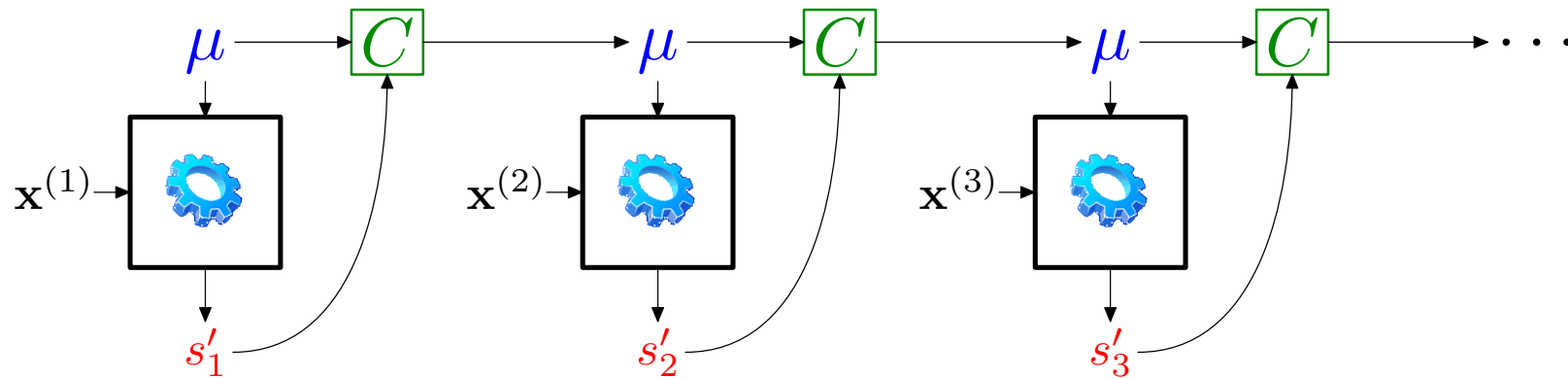
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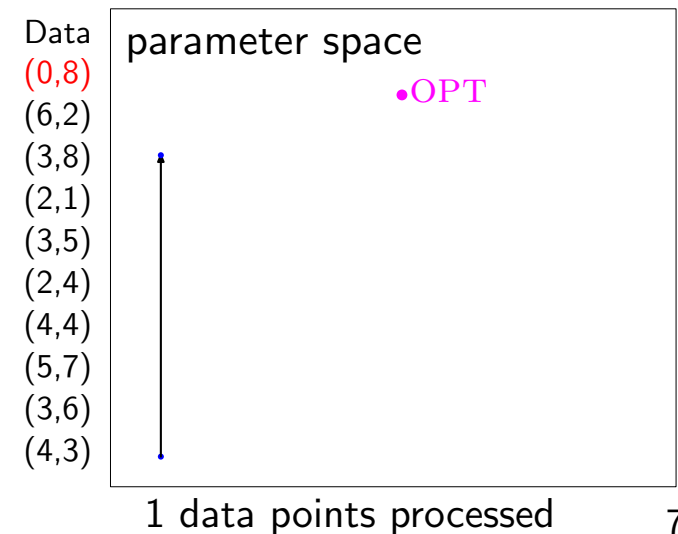
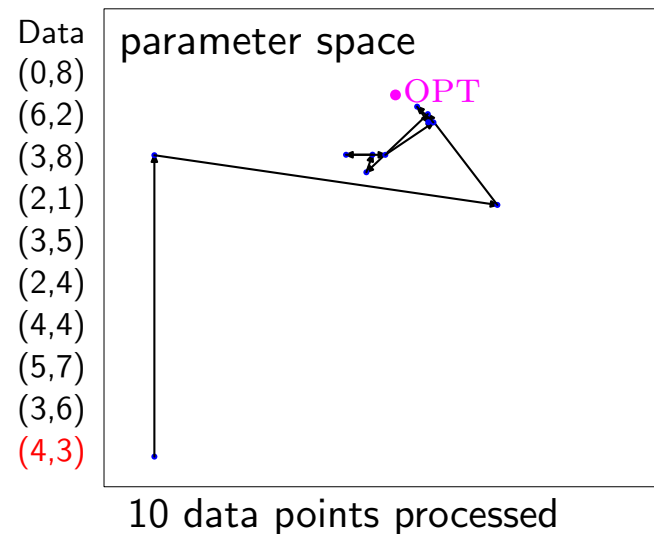
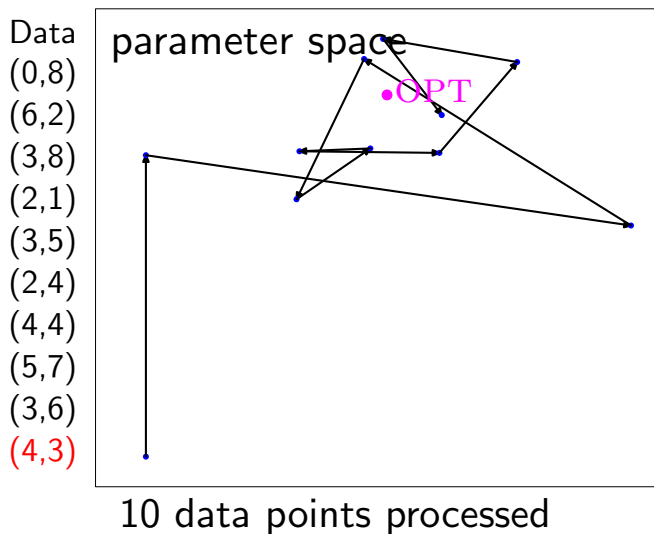


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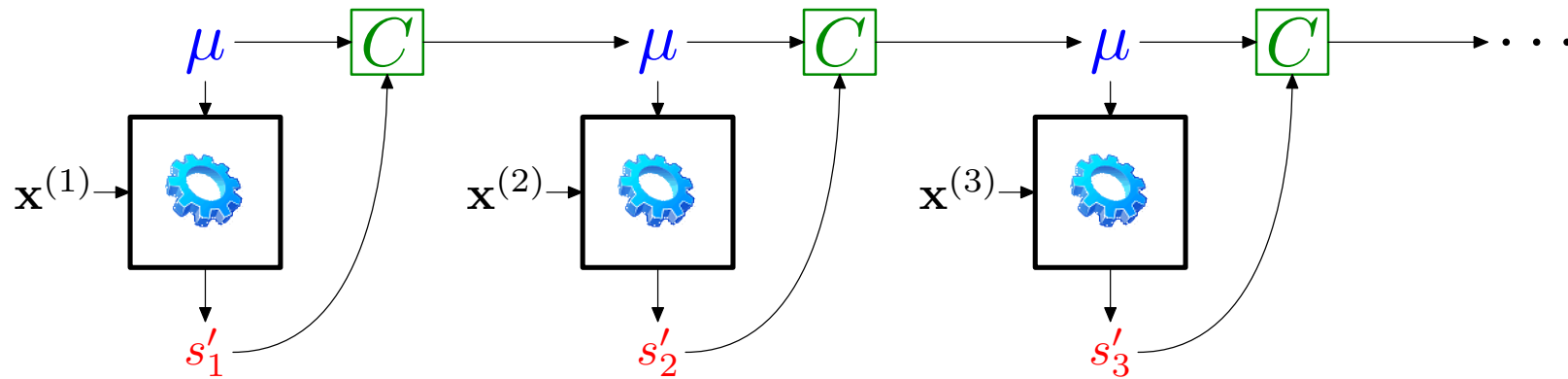
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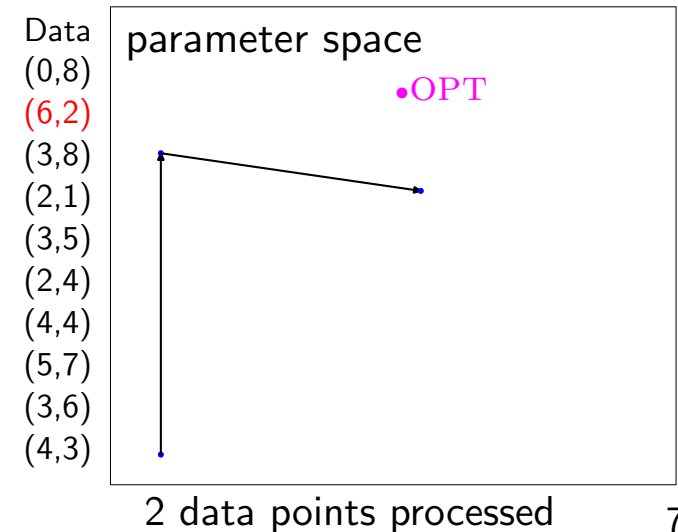
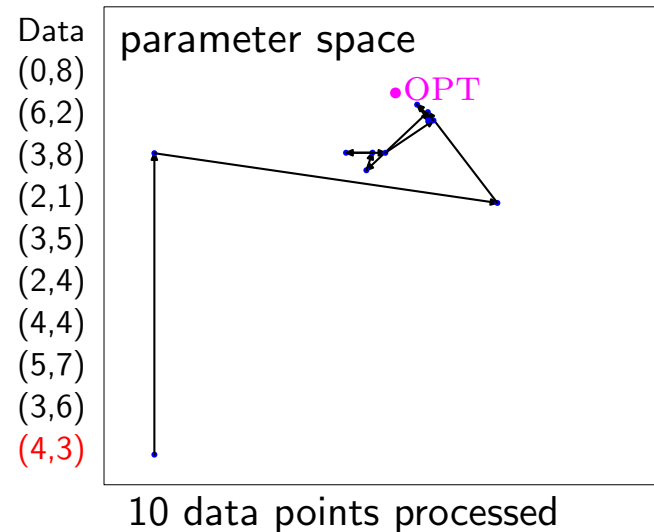
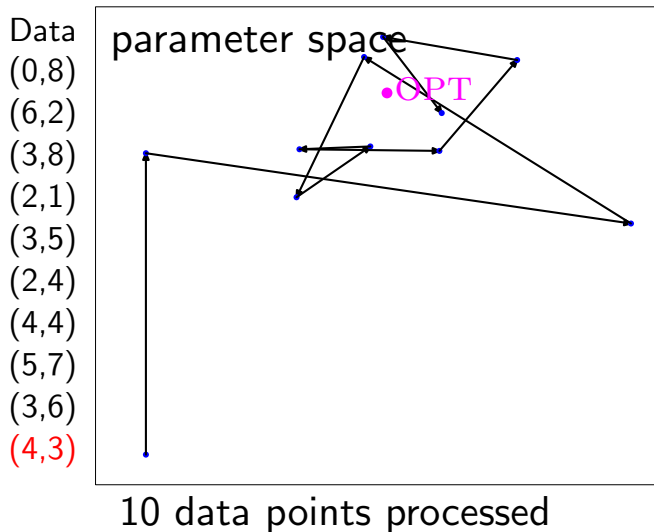


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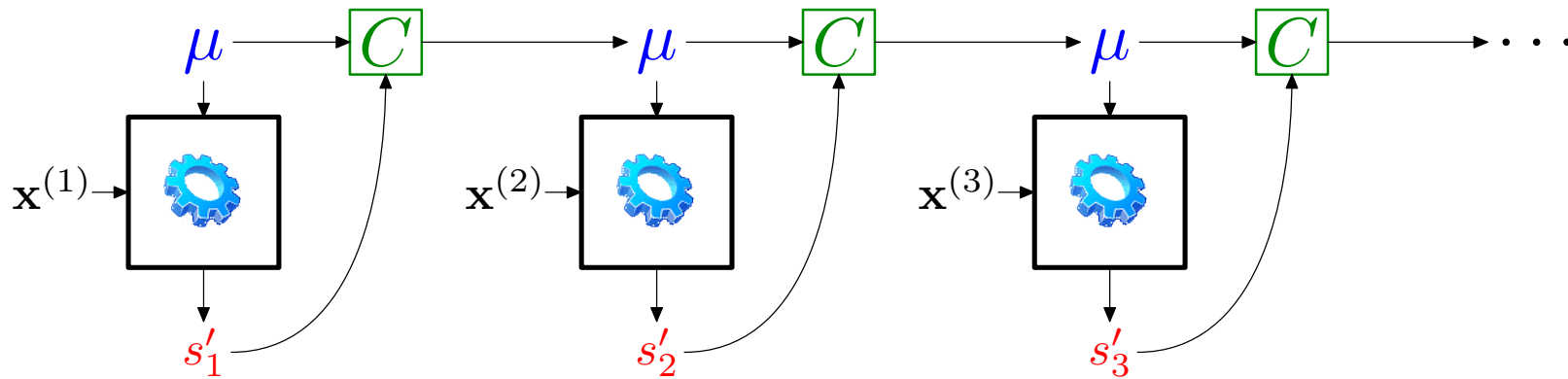
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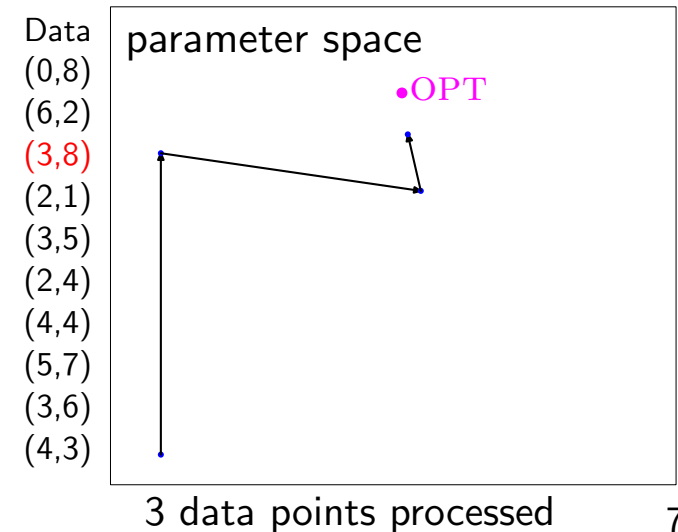
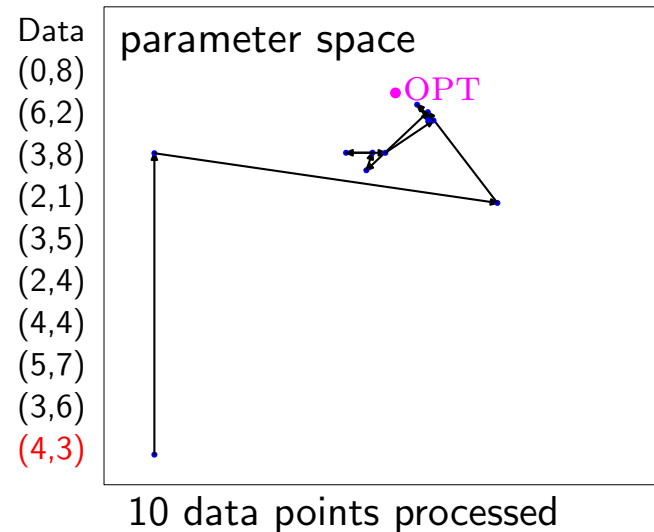
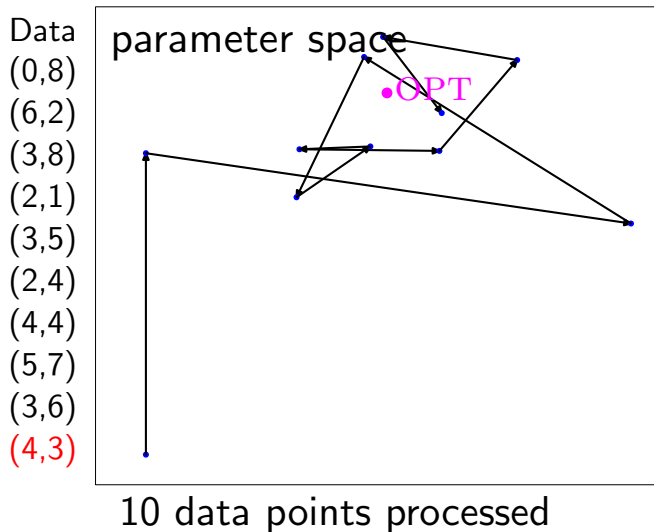


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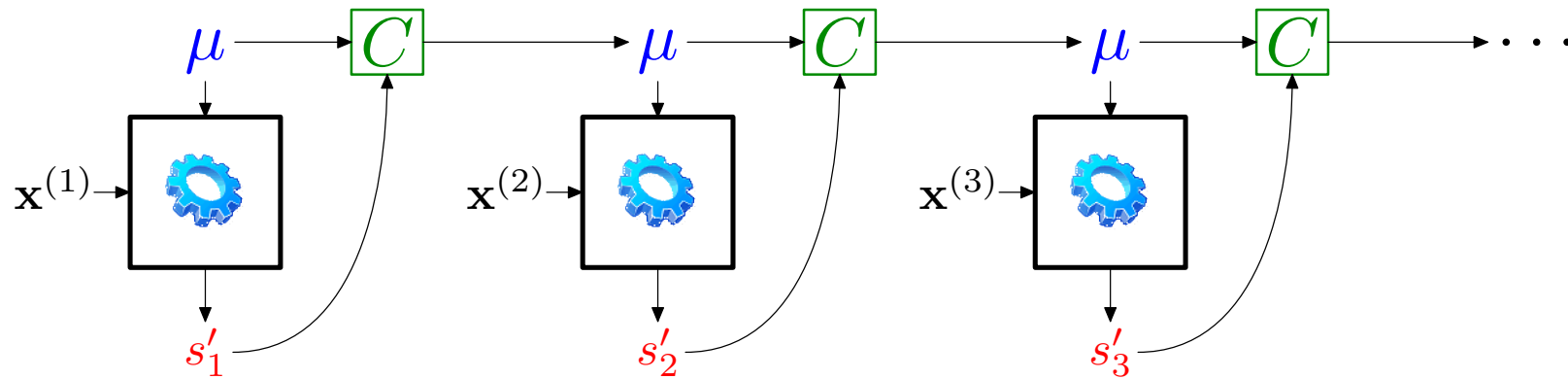
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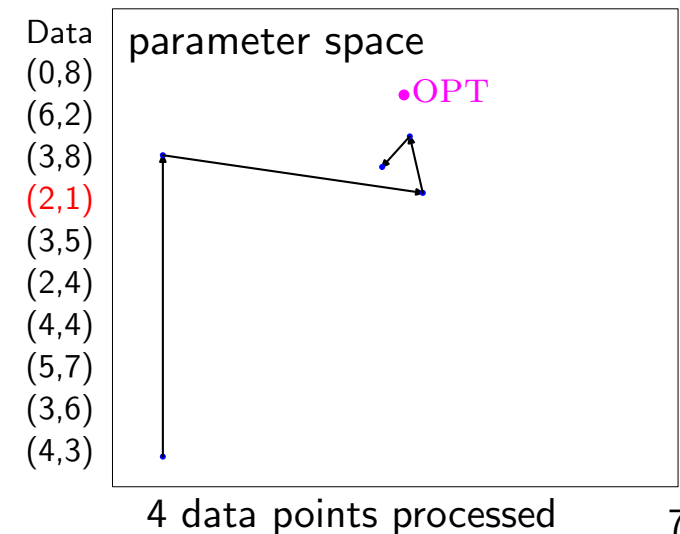
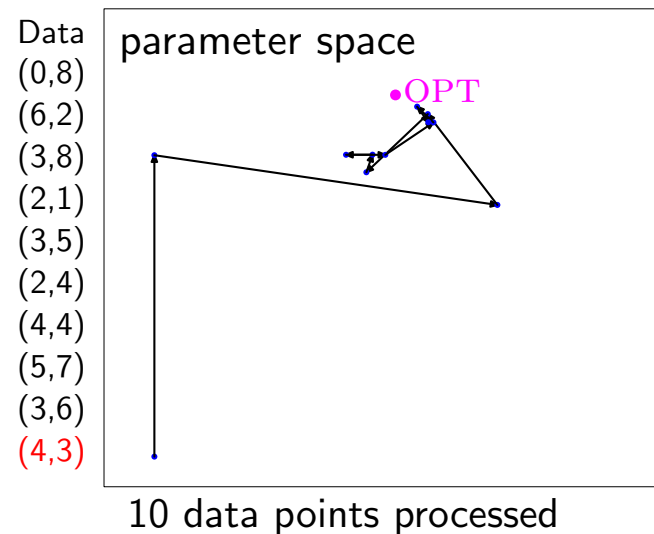
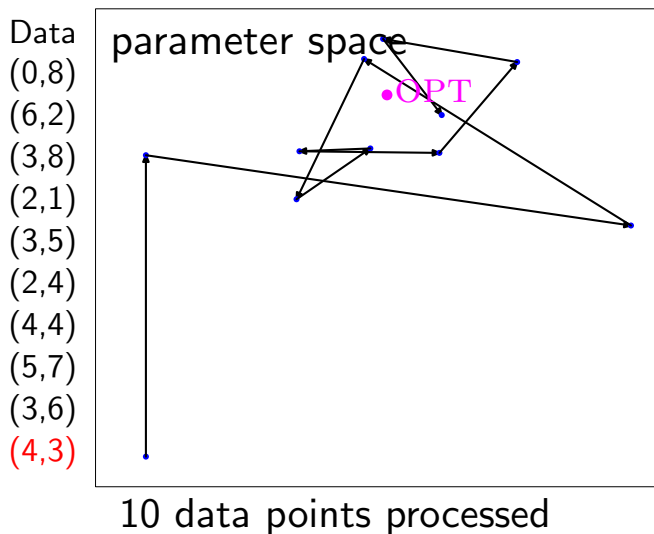


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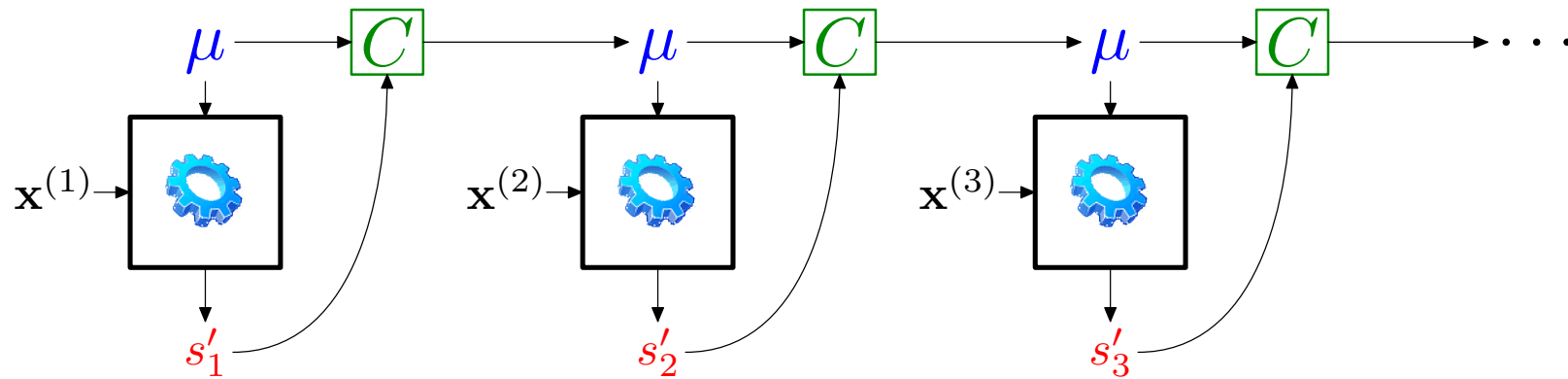
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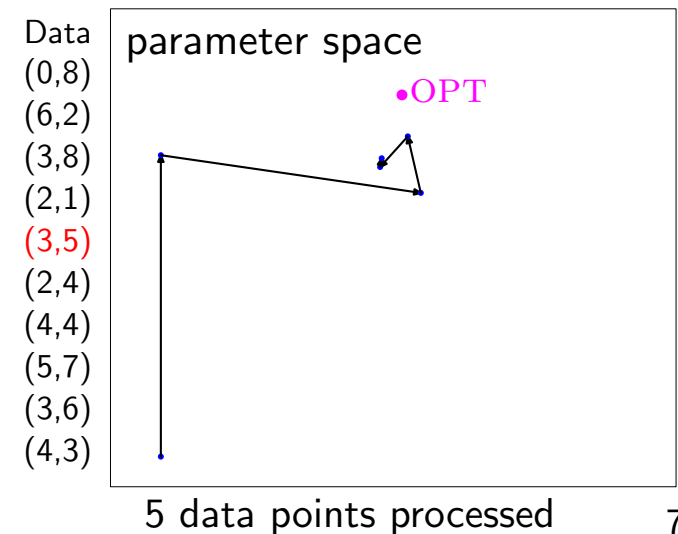
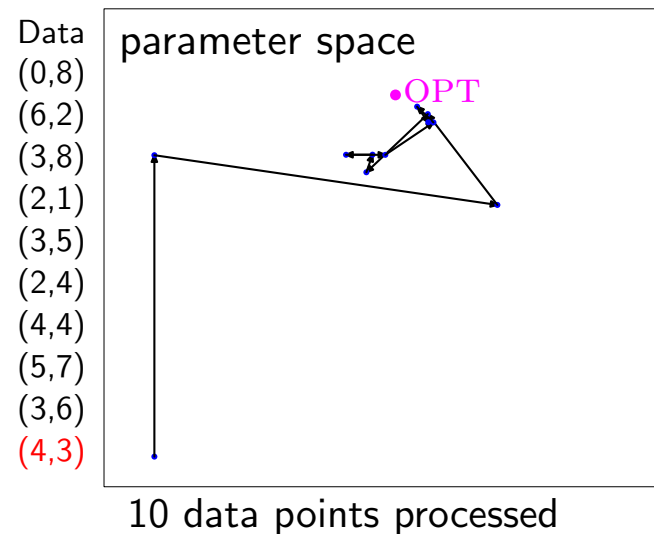
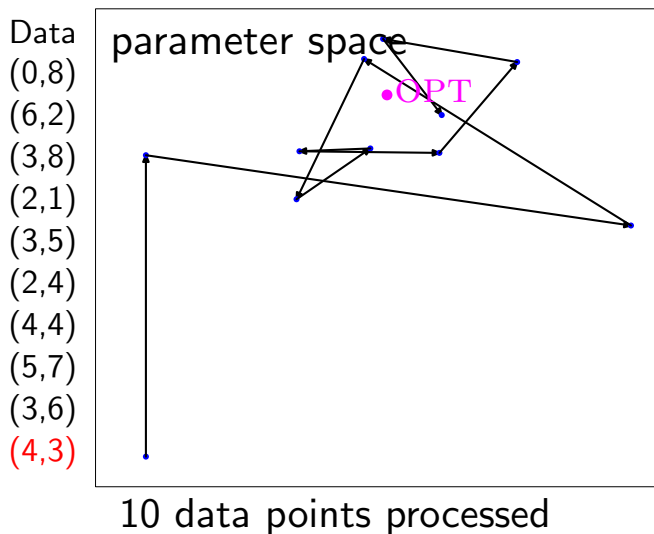


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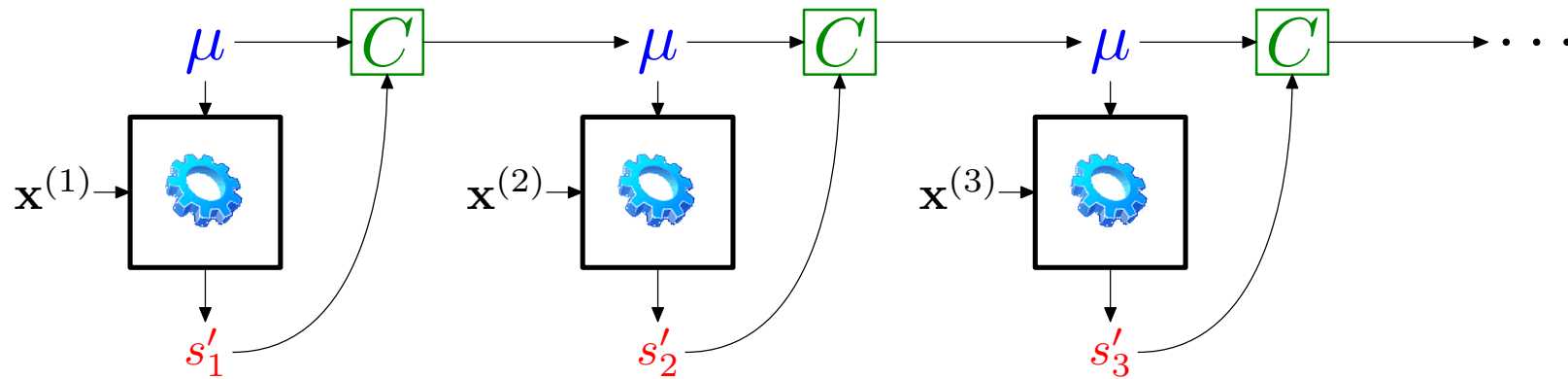
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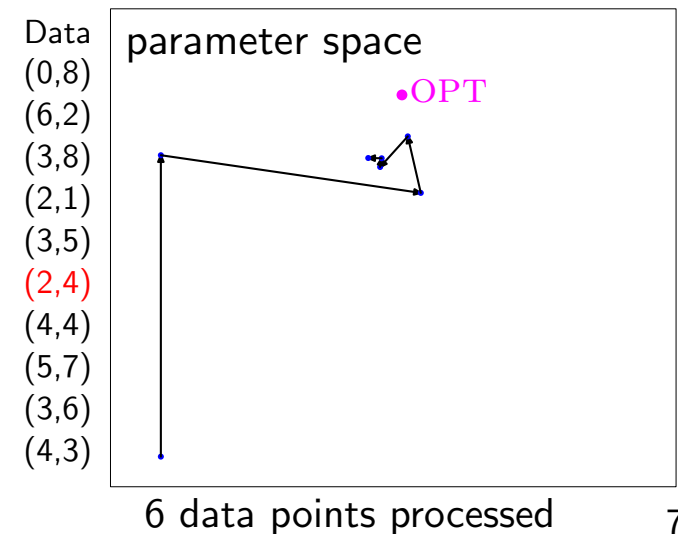
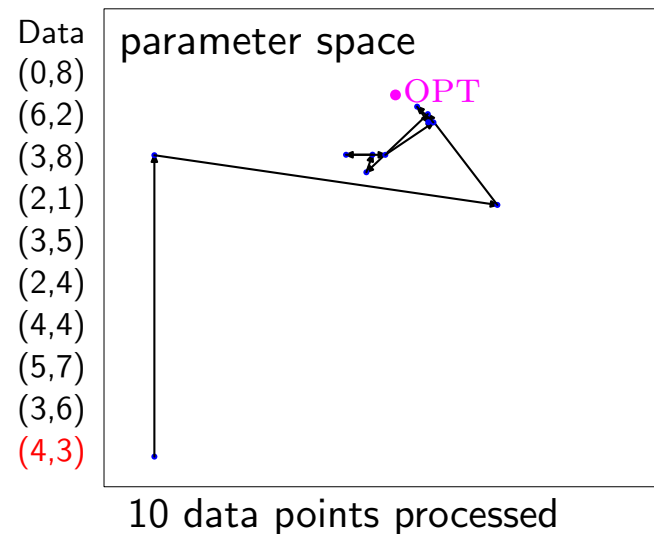
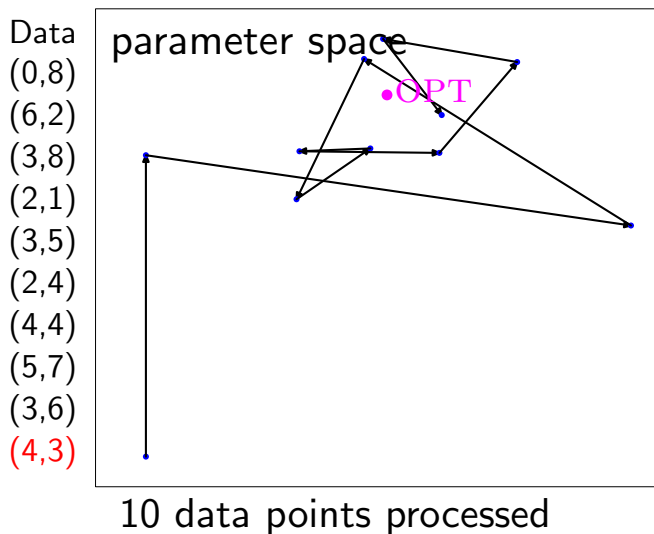


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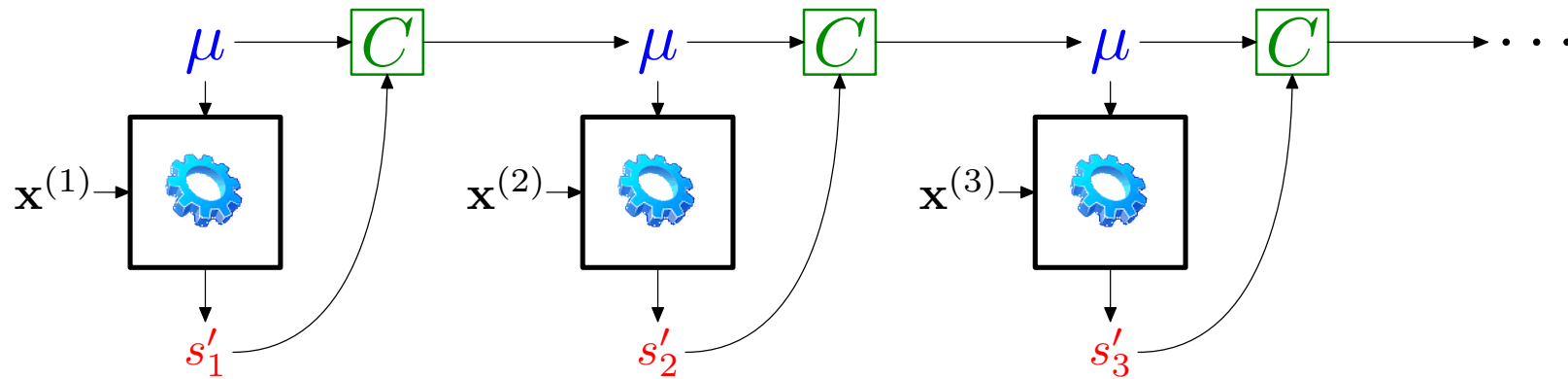
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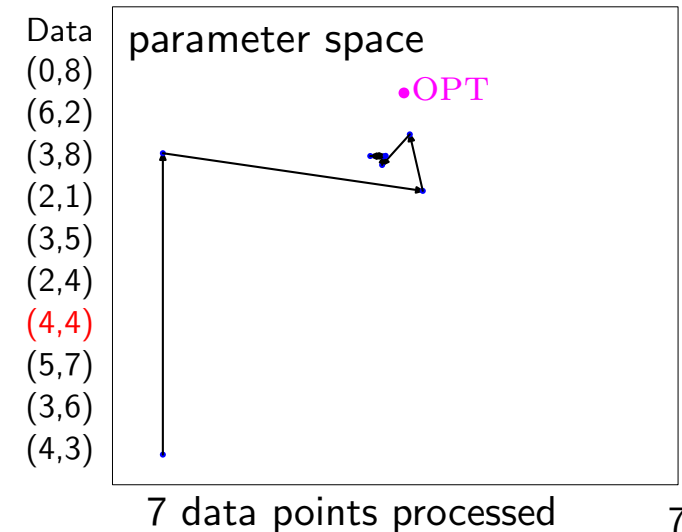
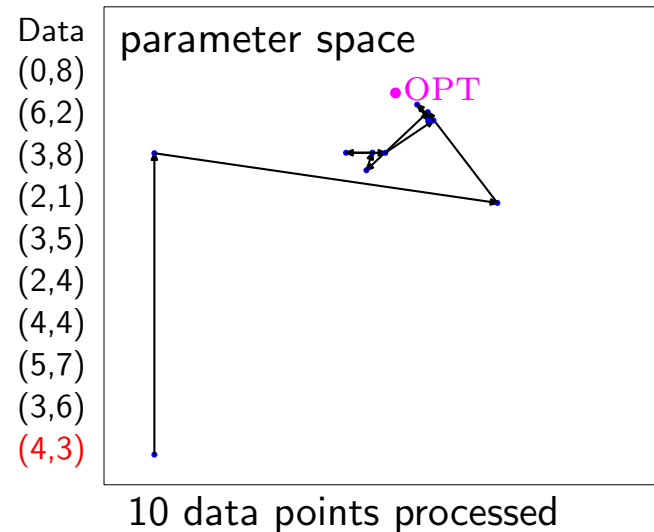
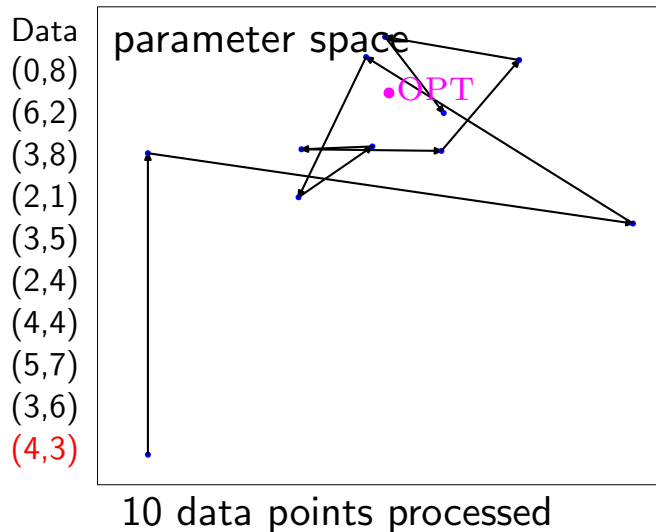


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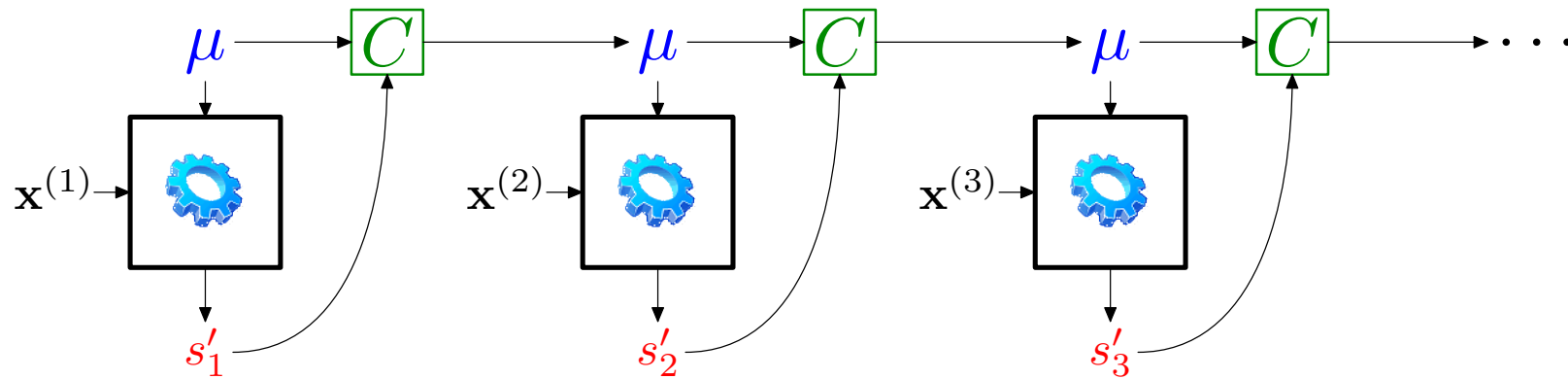
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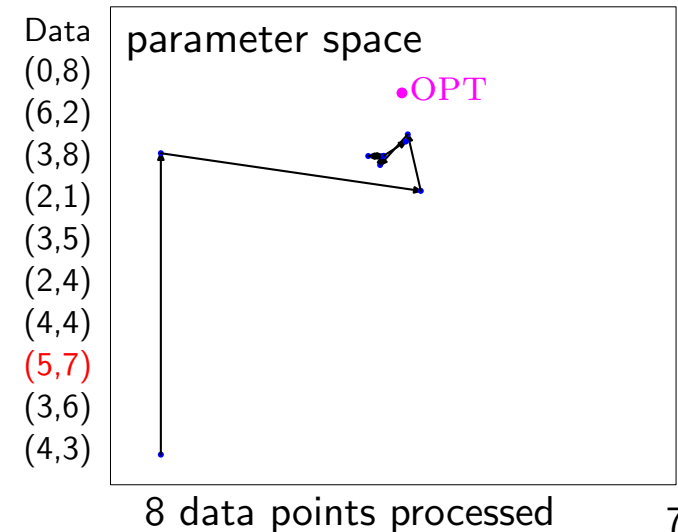
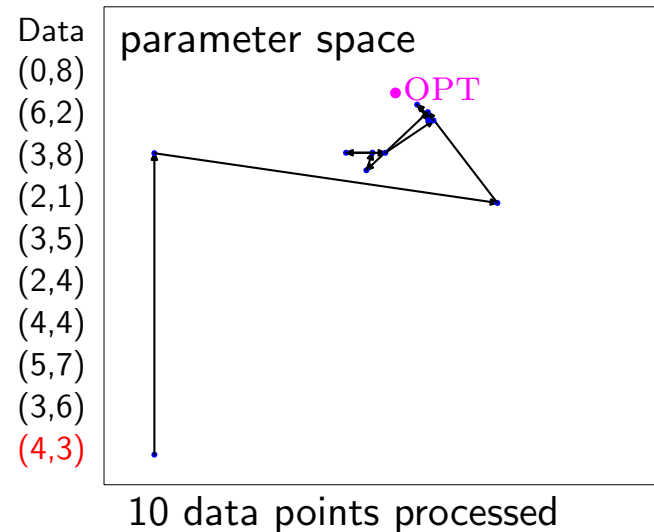
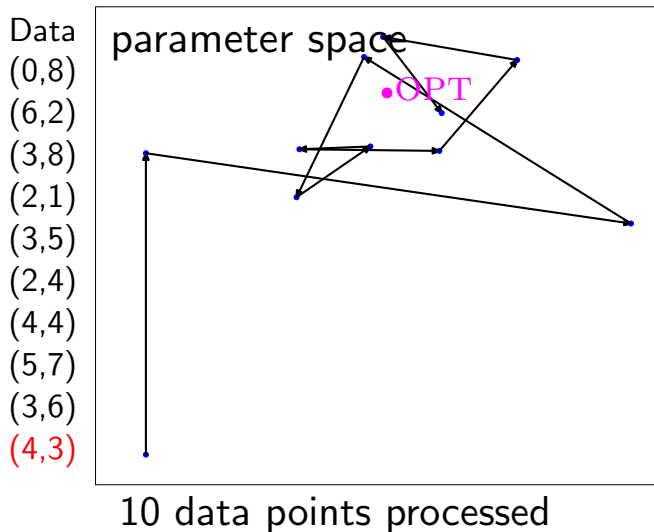


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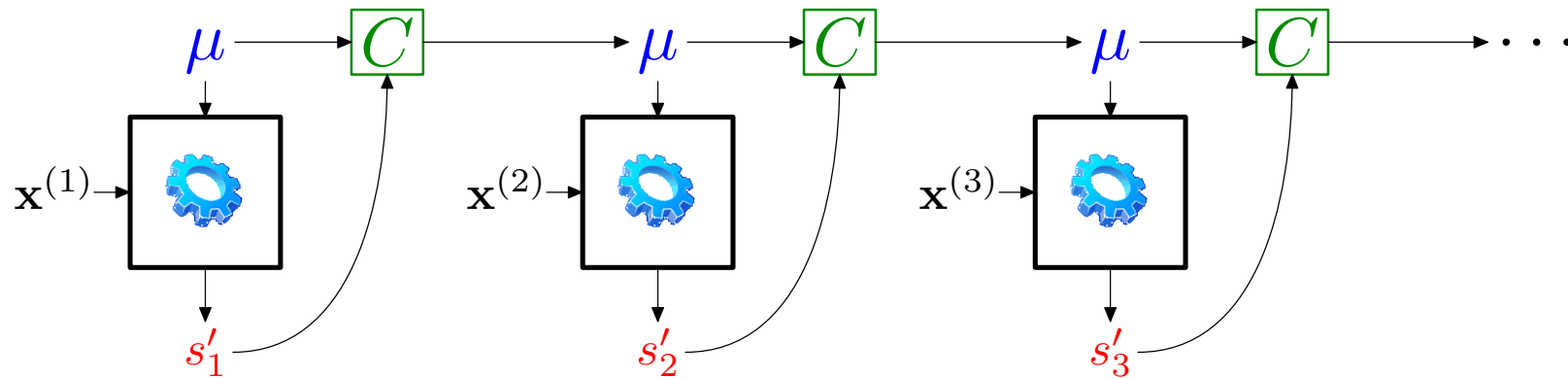
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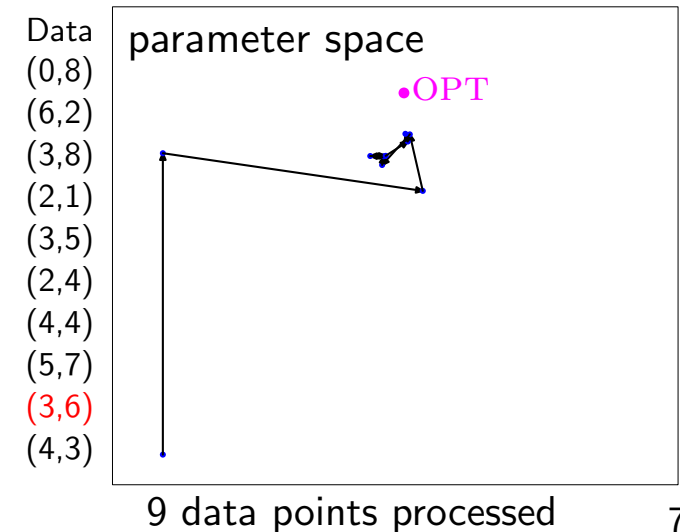
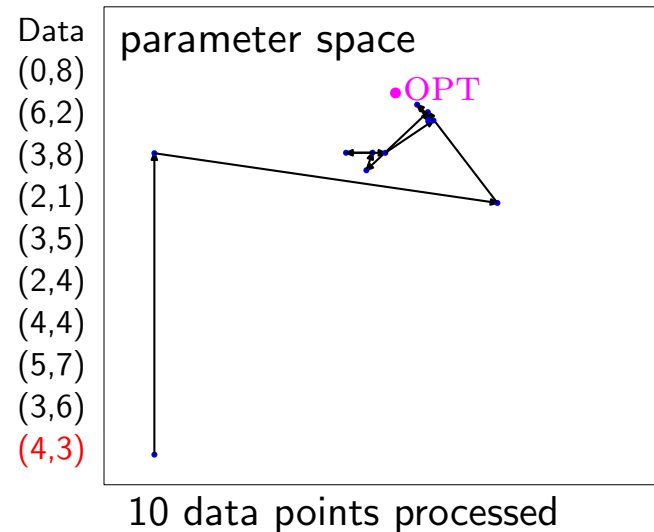
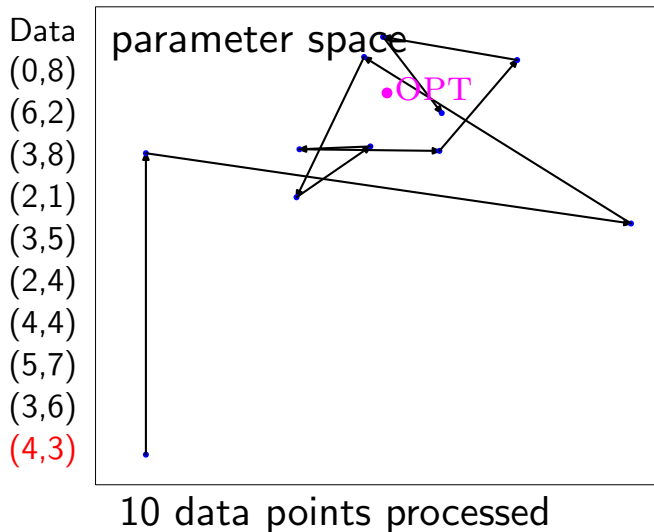


Combine old  $\mu$  and new  $s'_i$ :

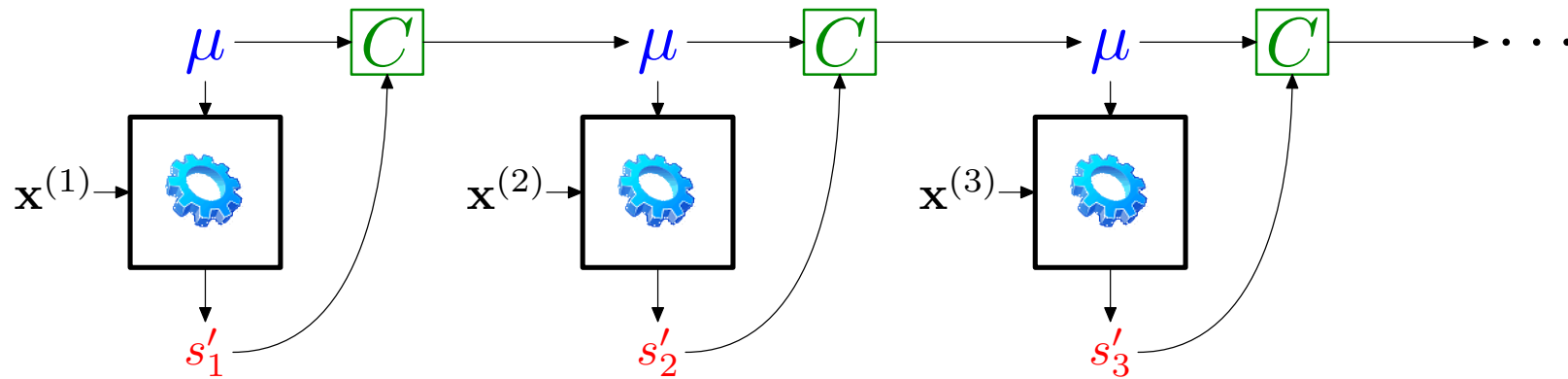
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

→  $\alpha = 1$  small updates, stable



# Optimization parameter 1 of 2: stepsize

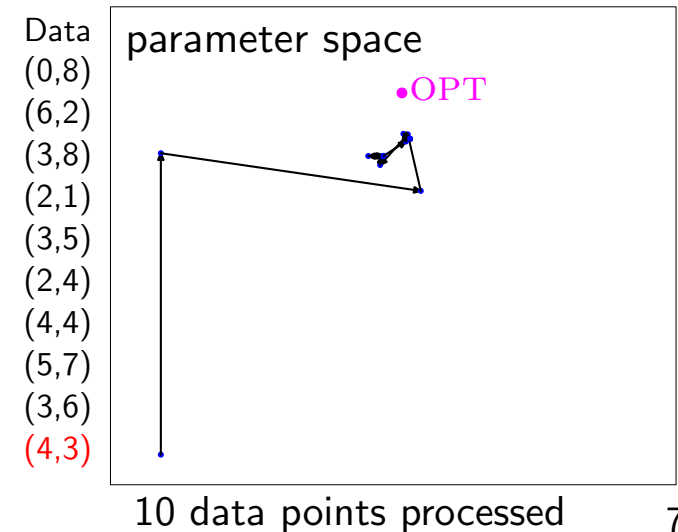
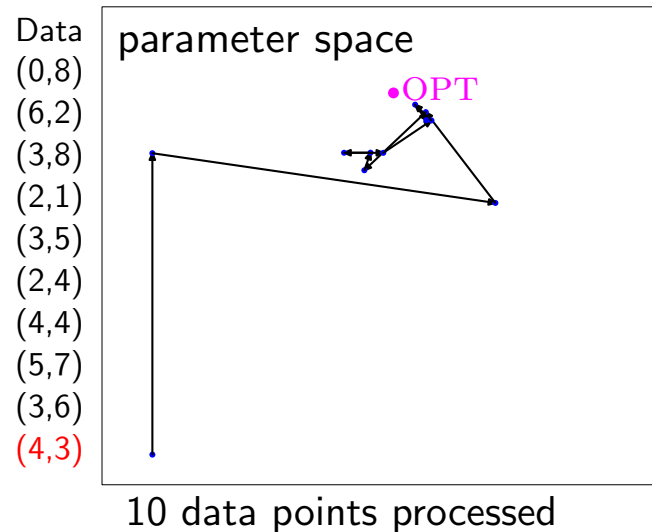
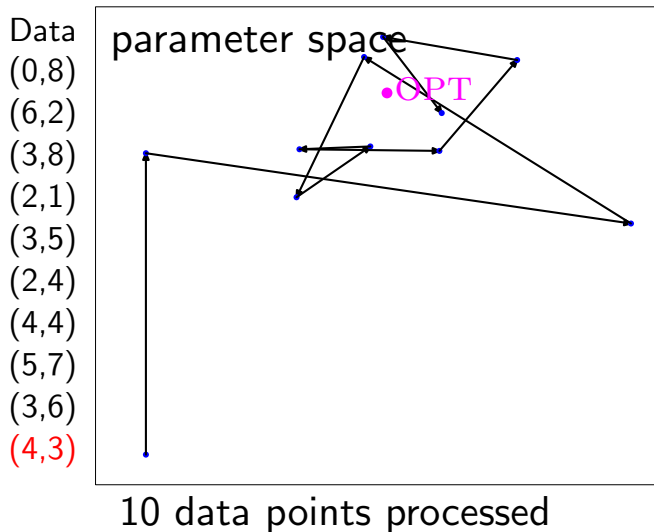


Combine old  $\mu$  and new  $s'_i$ :

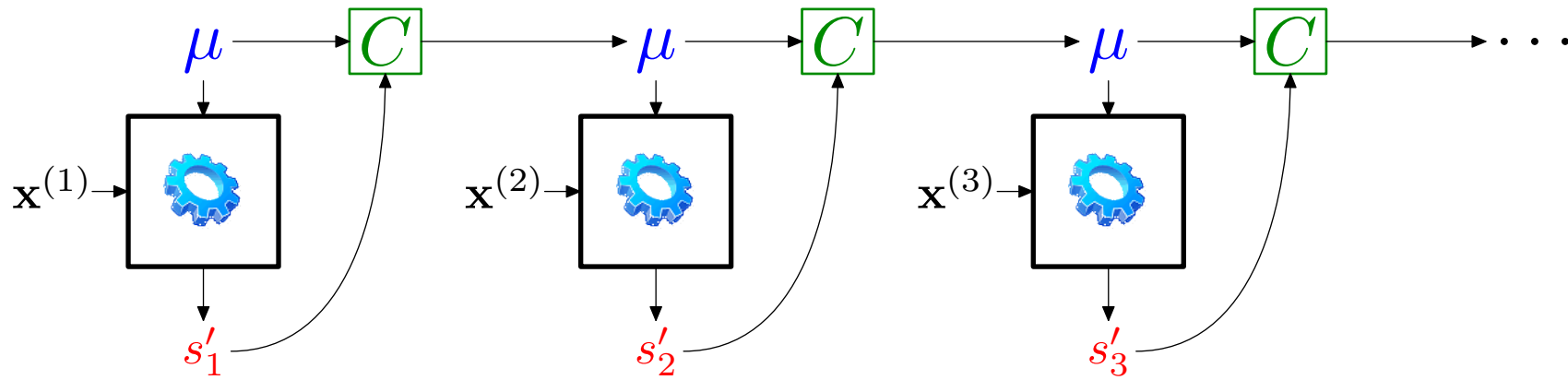
$$C(\mu, s'_i) = (1 - \eta_k)\mu + \eta_k s'_i, \quad \eta_k = \frac{1}{k^\alpha} \text{ on } k\text{-th update}$$

$\alpha = \frac{1}{2}$  ← large updates, unstable

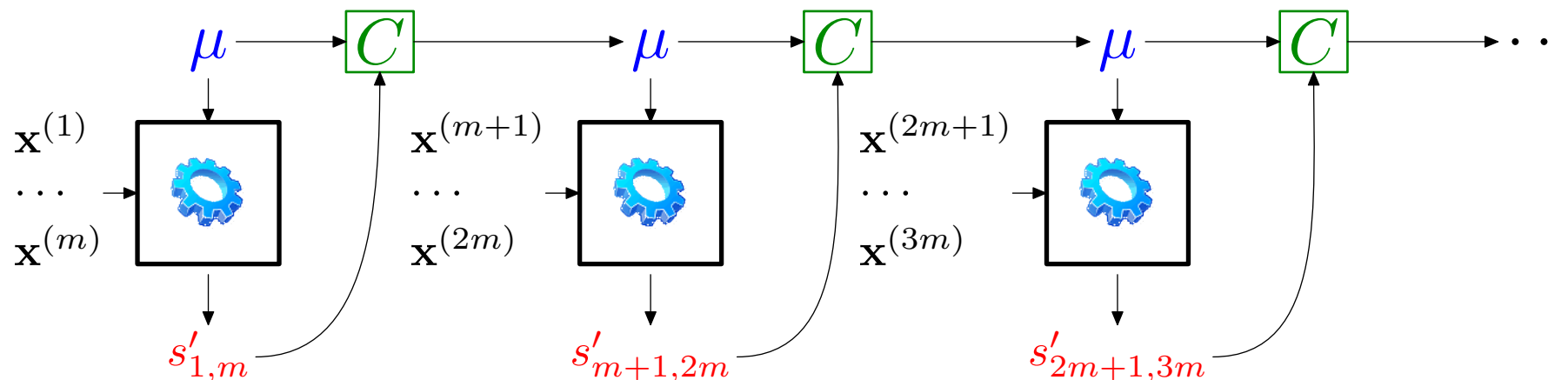
→  $\alpha = 1$  small updates, stable



# Optimization parameter 2 of 2: minibatch size

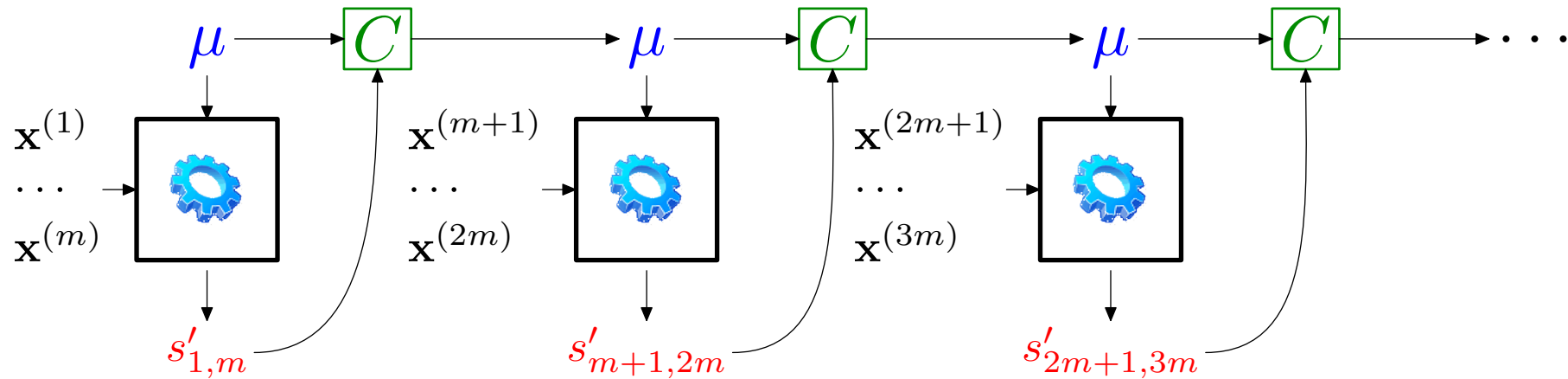


# Optimization parameter 2 of 2: minibatch size



$m = \text{size of a mini-batch}$

# Optimization parameter 2 of 2: minibatch size



$m =$  size of a mini-batch

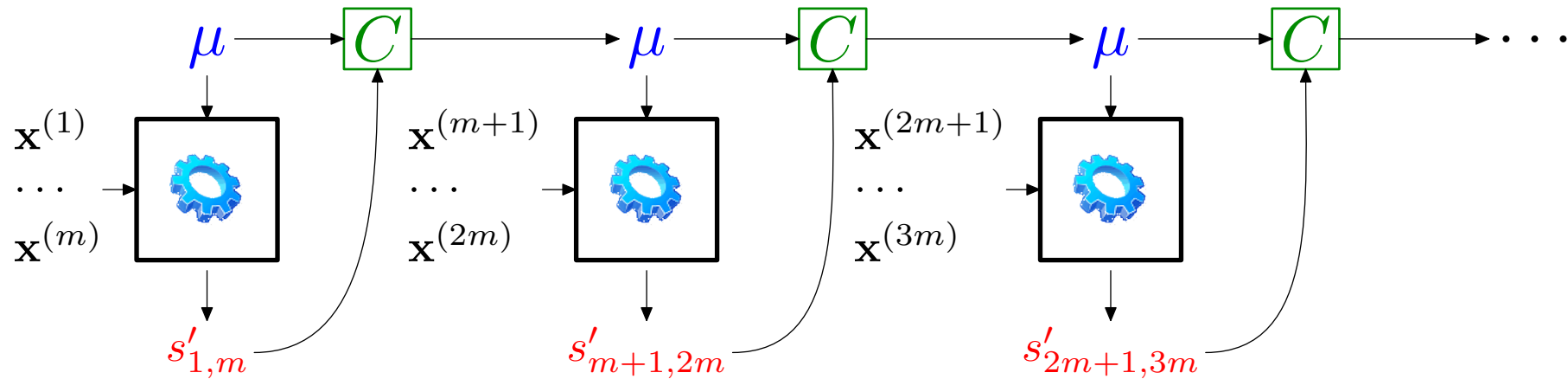
$m = 1$

frequent updates, unstable

$m = n$

infrequent updates, stable

# Optimization parameter 2 of 2: minibatch size



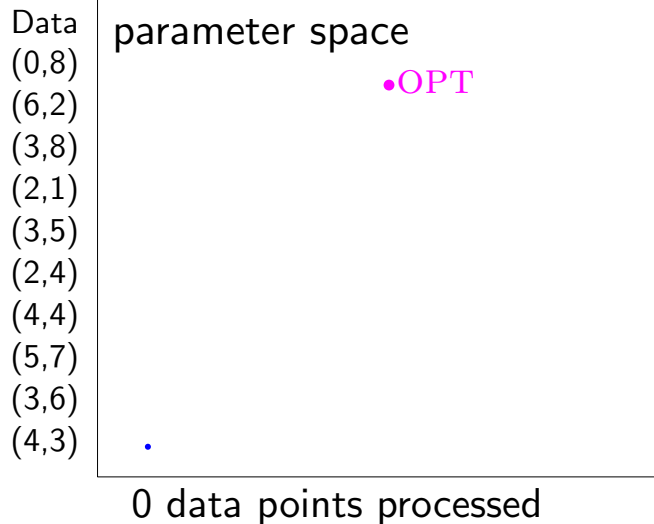
$m =$  size of a mini-batch

$m = 1$

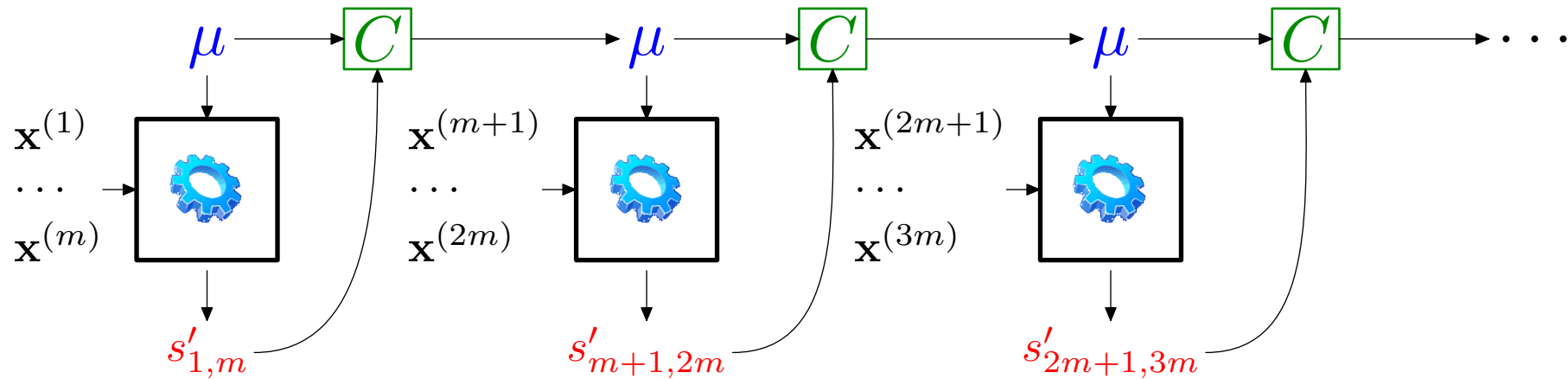
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



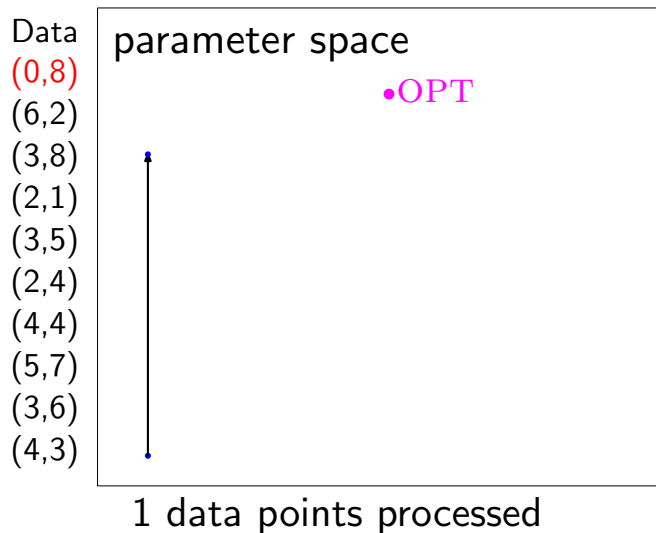
$m = \text{size of a mini-batch}$

$m = 1$

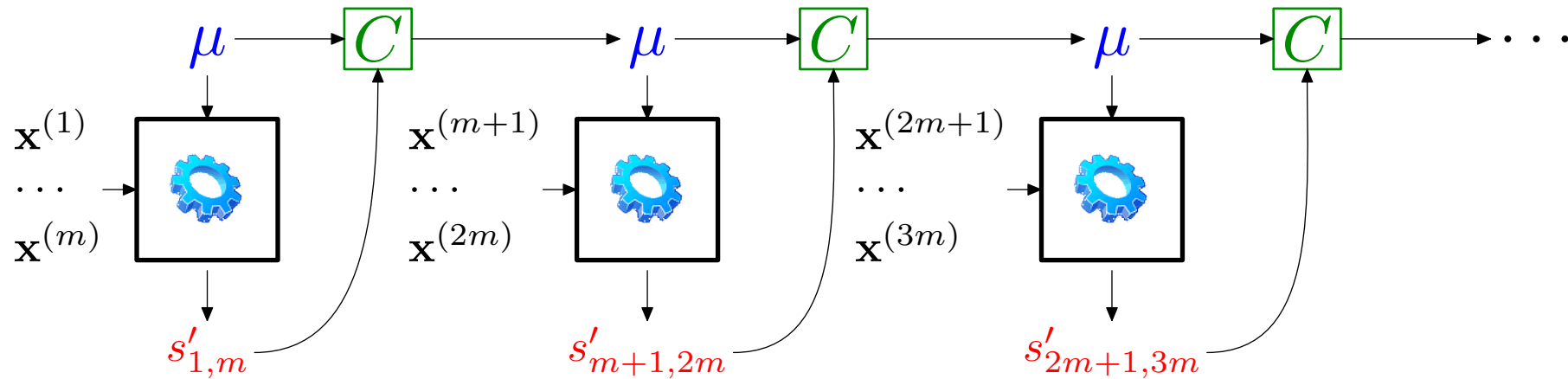
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



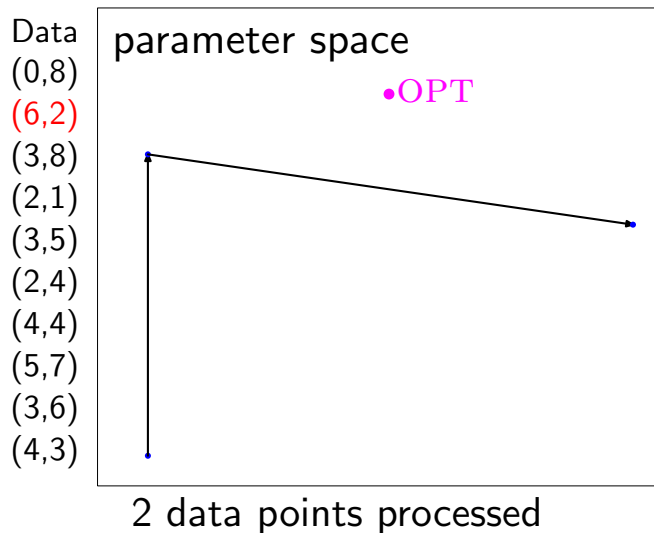
$m =$  size of a mini-batch

$m = 1$

$m = n$

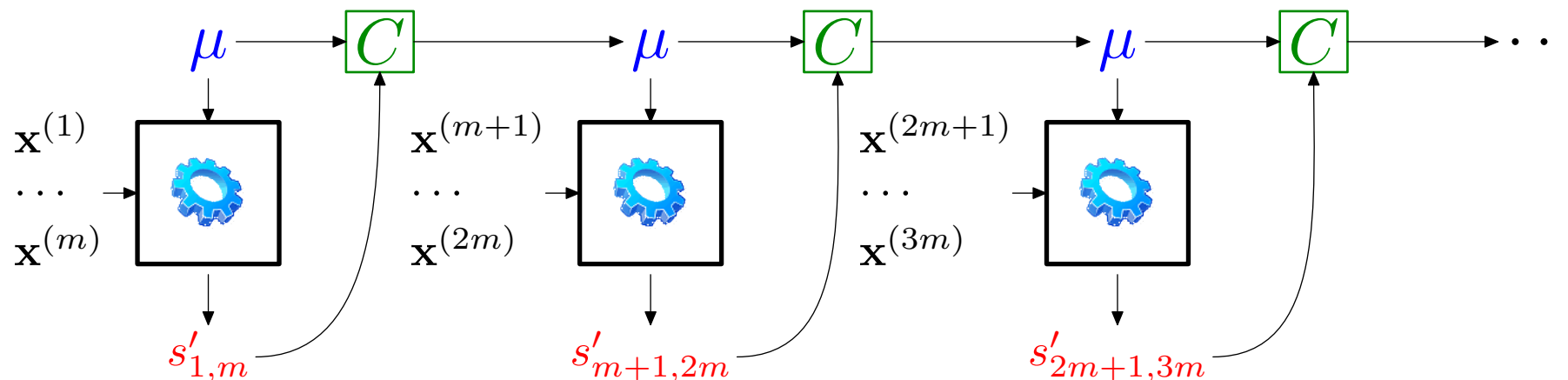
frequent updates, unstable

infrequent updates, stable





# Optimization parameter 2 of 2: minibatch size



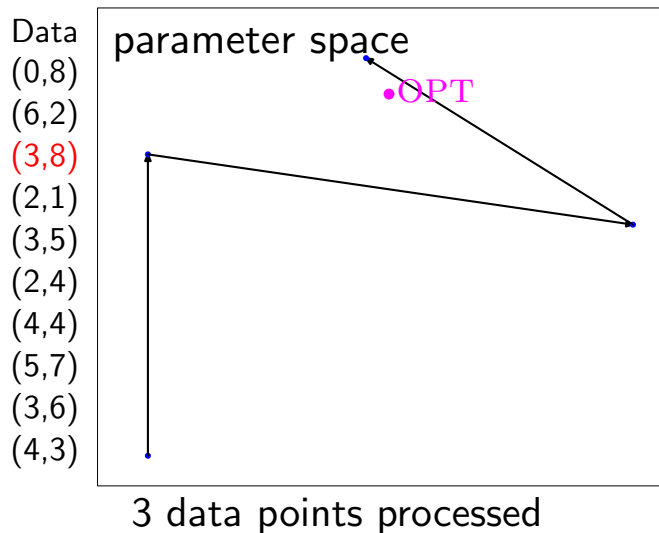
$m = \text{size of a mini-batch}$

$m = 1$

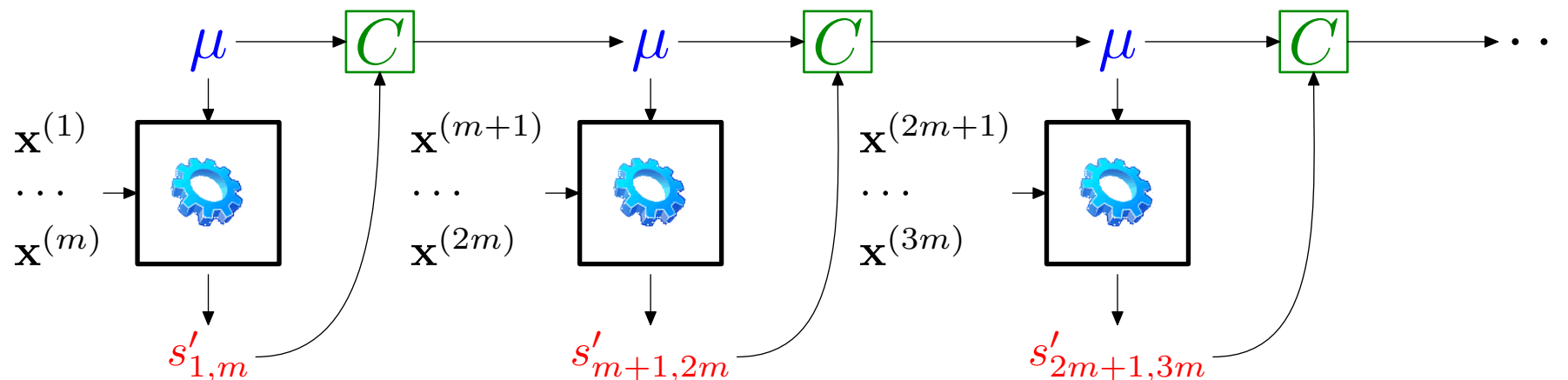
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



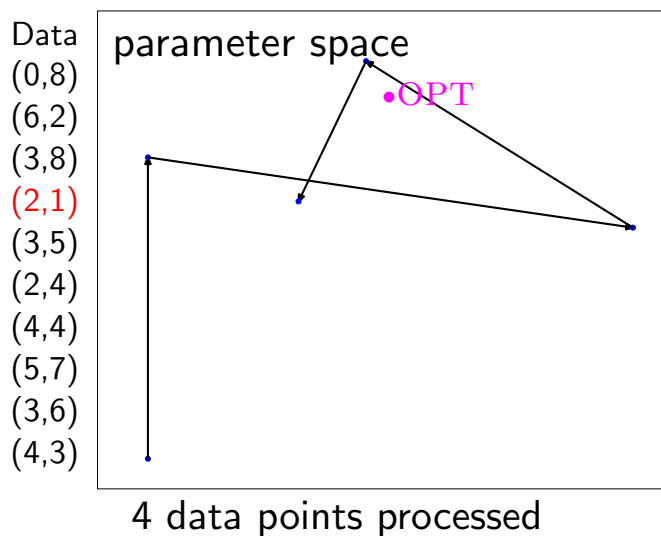
$m = \text{size of a mini-batch}$

$m = 1$

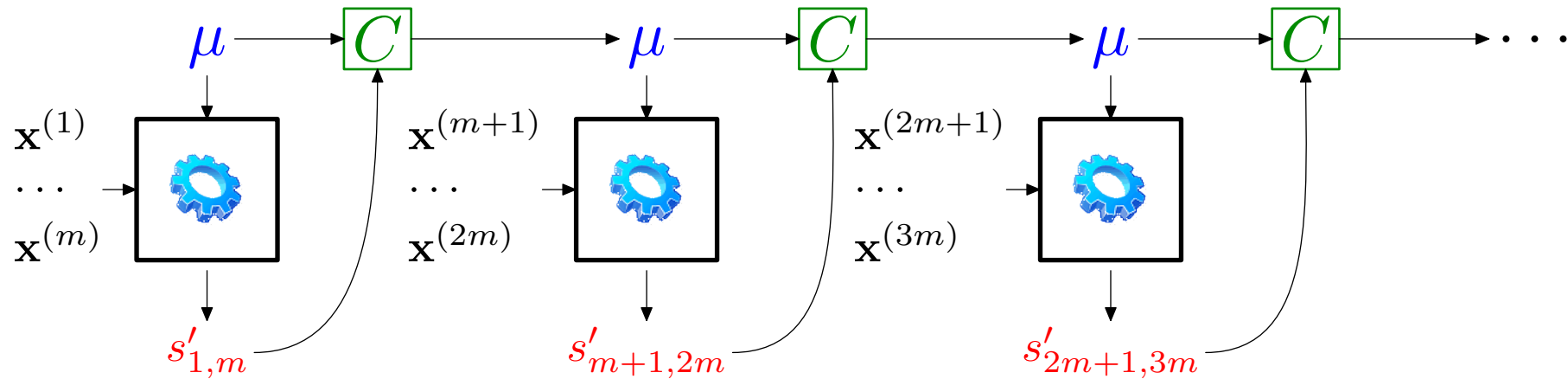
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



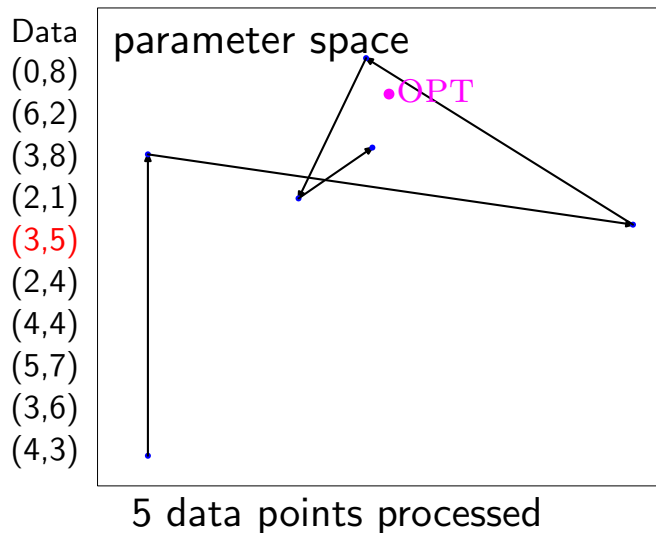
$m = \text{size of a mini-batch}$

$m = 1$

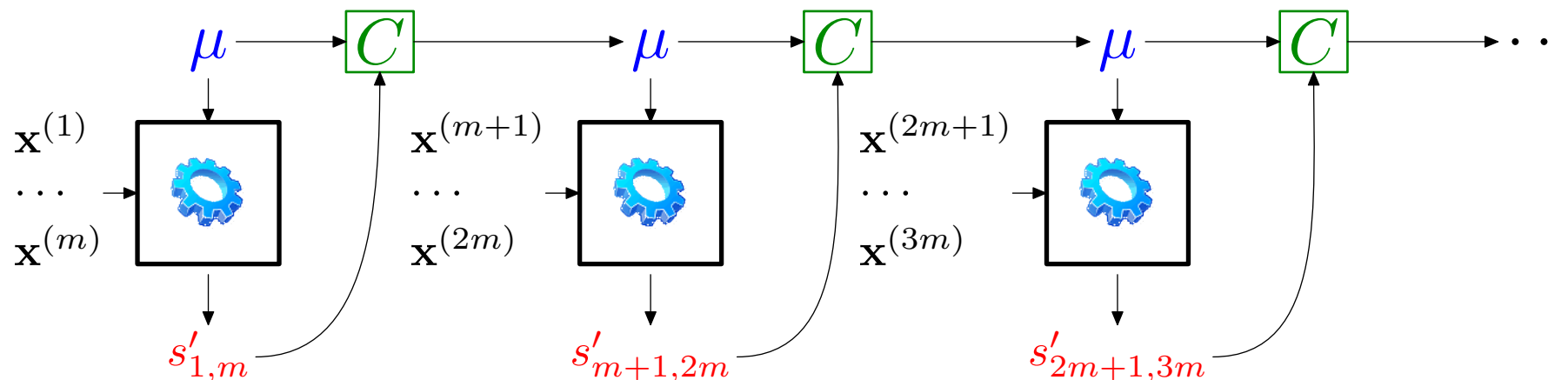
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



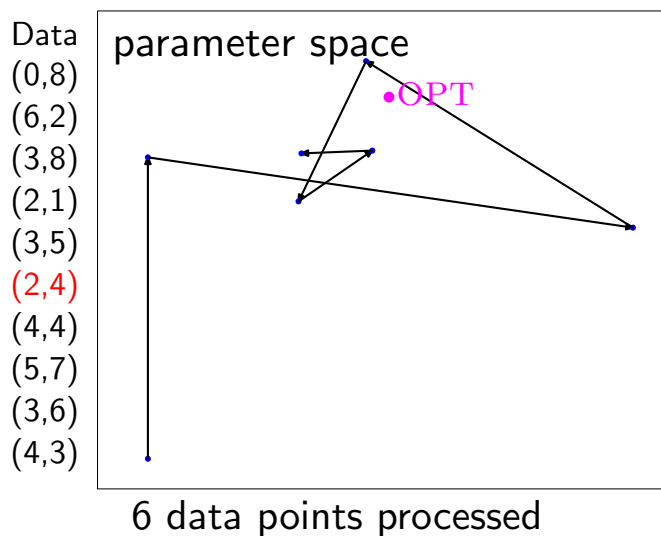
$m = \text{size of a mini-batch}$

$m = 1$

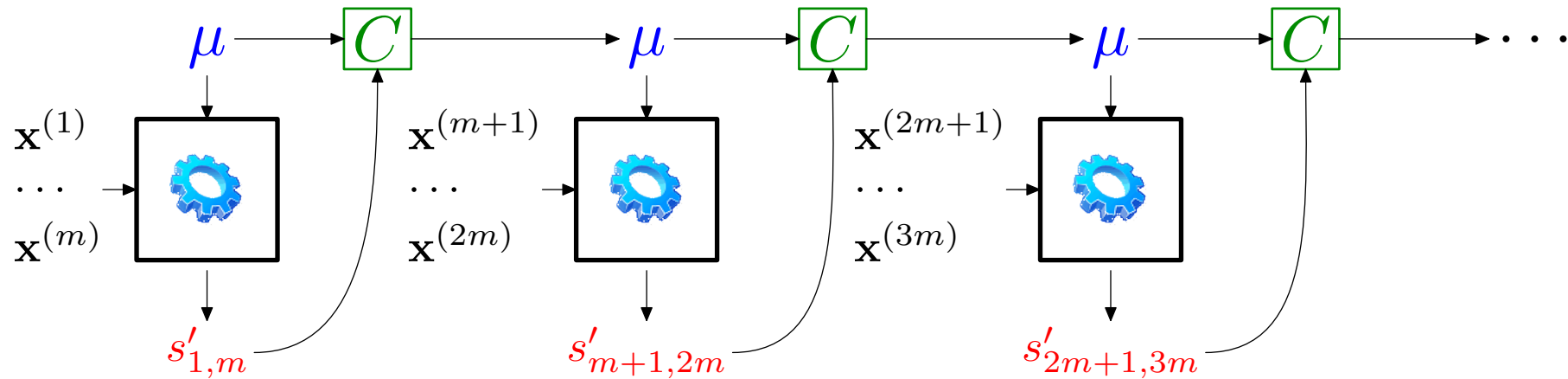
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



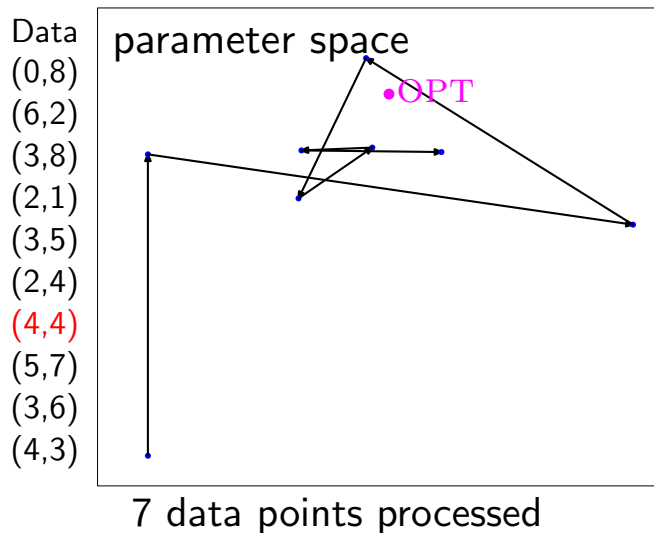
$m = \text{size of a mini-batch}$

$m = 1$

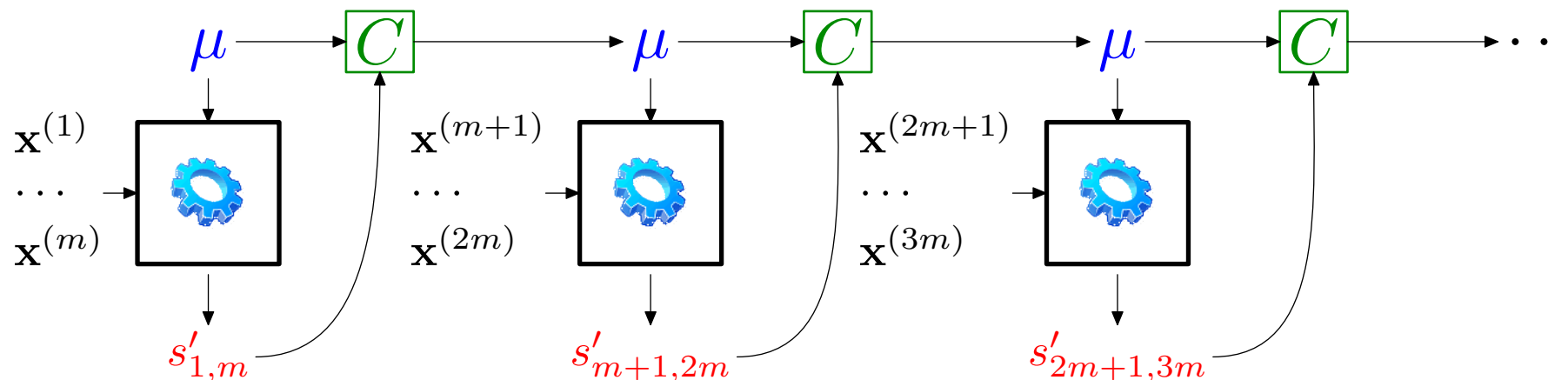
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



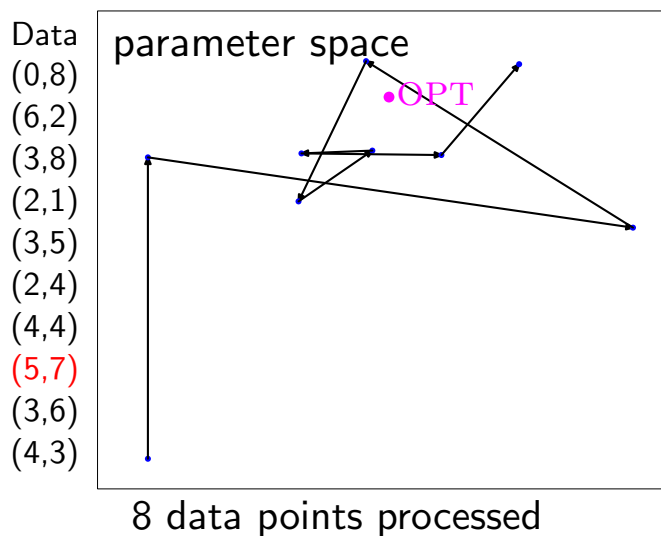
$m = \text{size of a mini-batch}$

$m = 1$

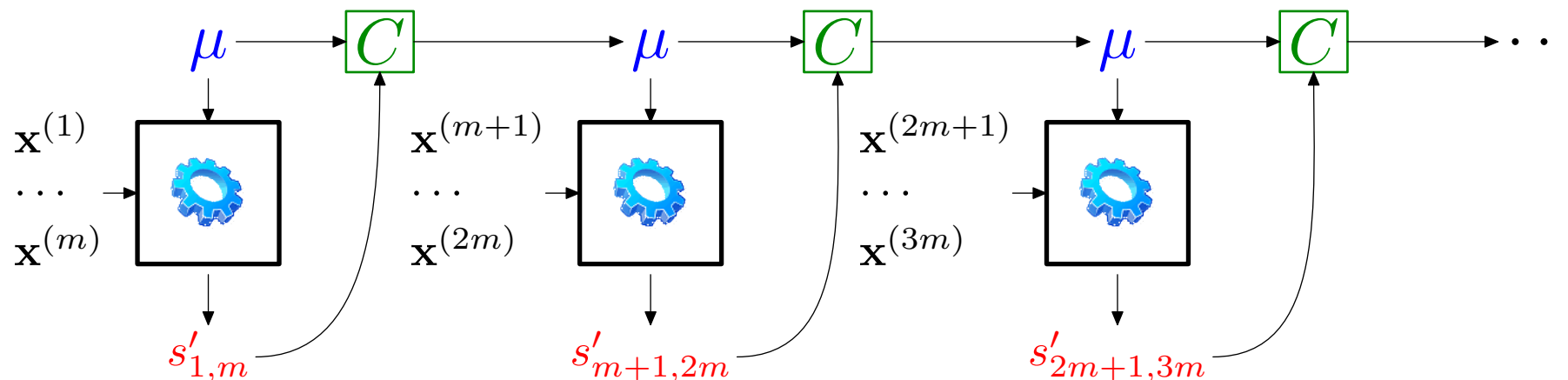
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



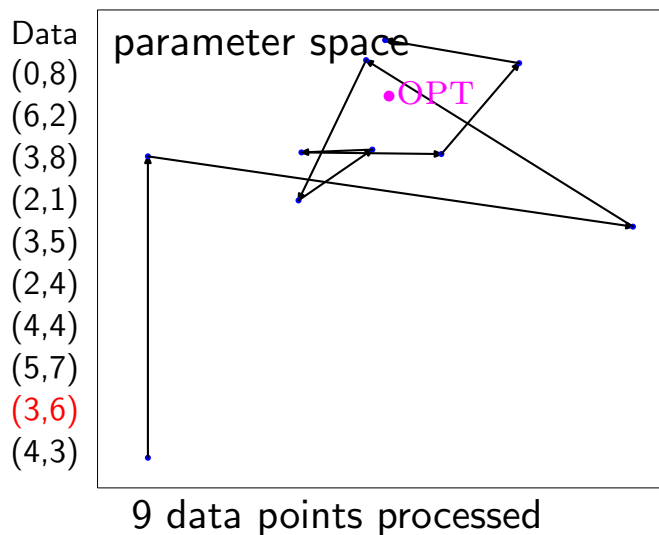
$m = \text{size of a mini-batch}$

$m = 1$

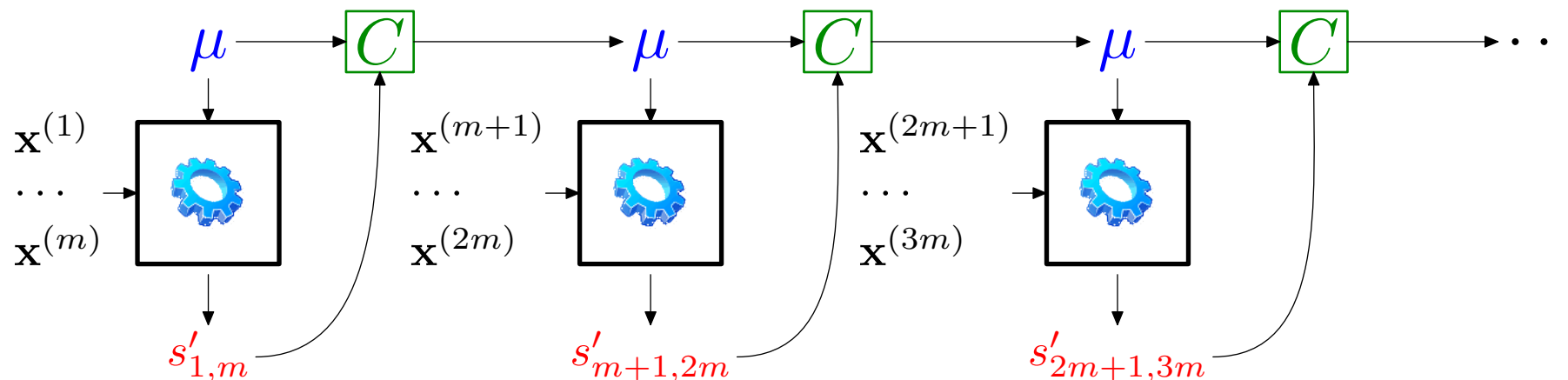
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



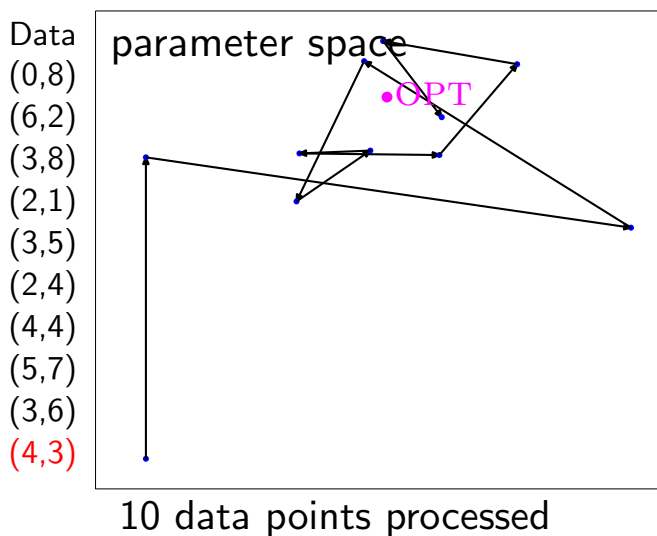
$m = \text{size of a mini-batch}$

$m = 1$

$m = n$

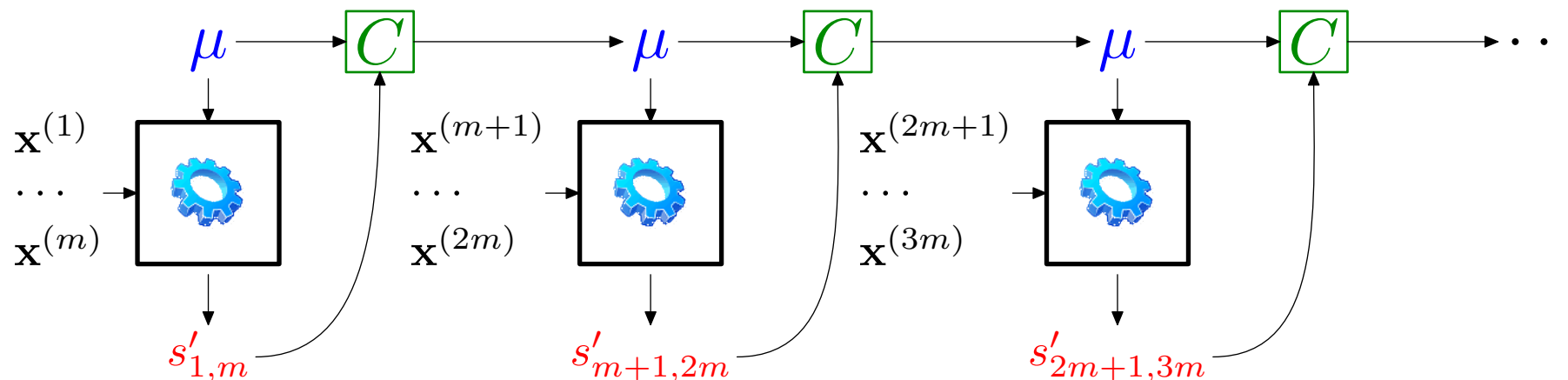
frequent updates, unstable

infrequent updates, stable





# Optimization parameter 2 of 2: minibatch size



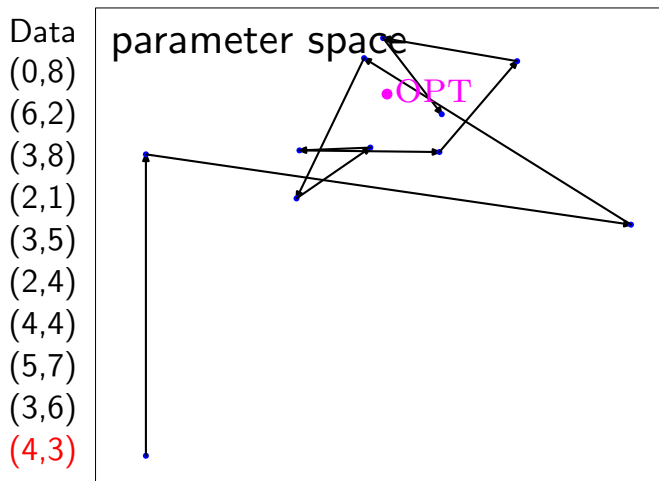
$m = \text{size of a mini-batch}$

$m = 1$

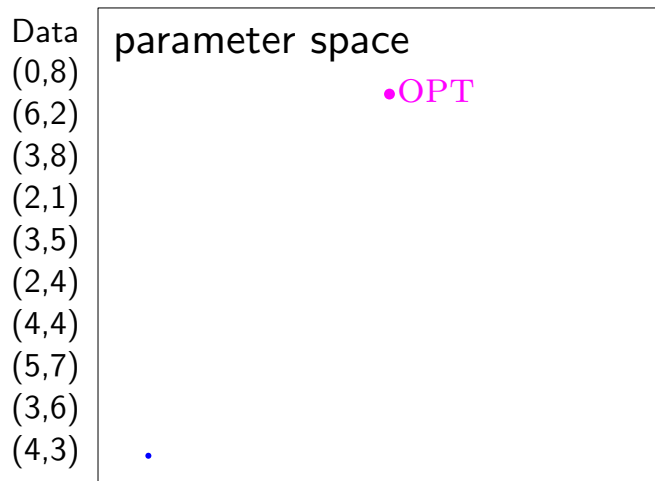
$m = n$

frequent updates, unstable

infrequent updates, stable

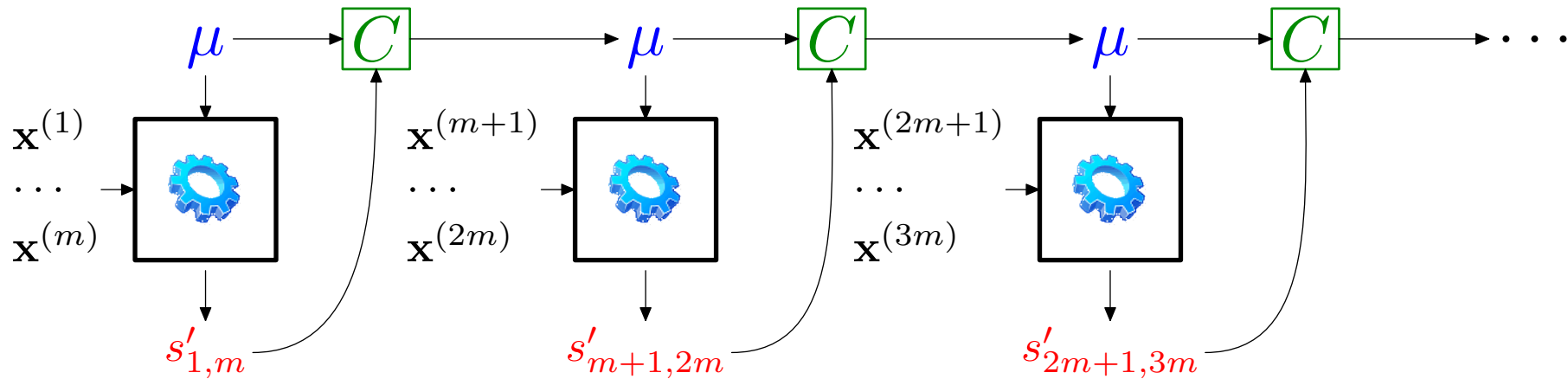


10 data points processed



0 data points processed

# Optimization parameter 2 of 2: minibatch size



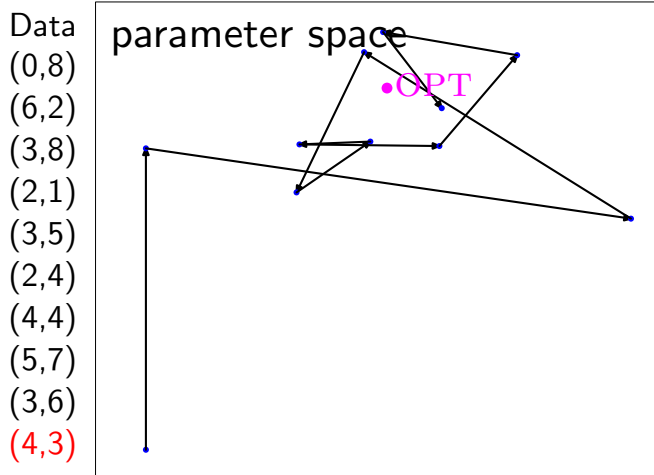
$m = \text{size of a mini-batch}$

$m = 1$

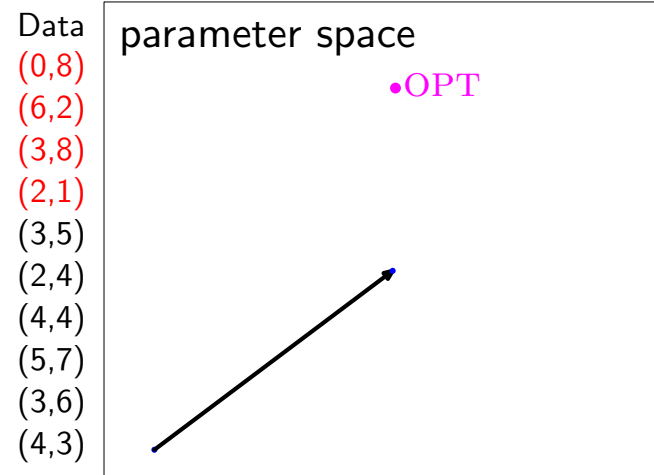
$m = n$

frequent updates, unstable

infrequent updates, stable

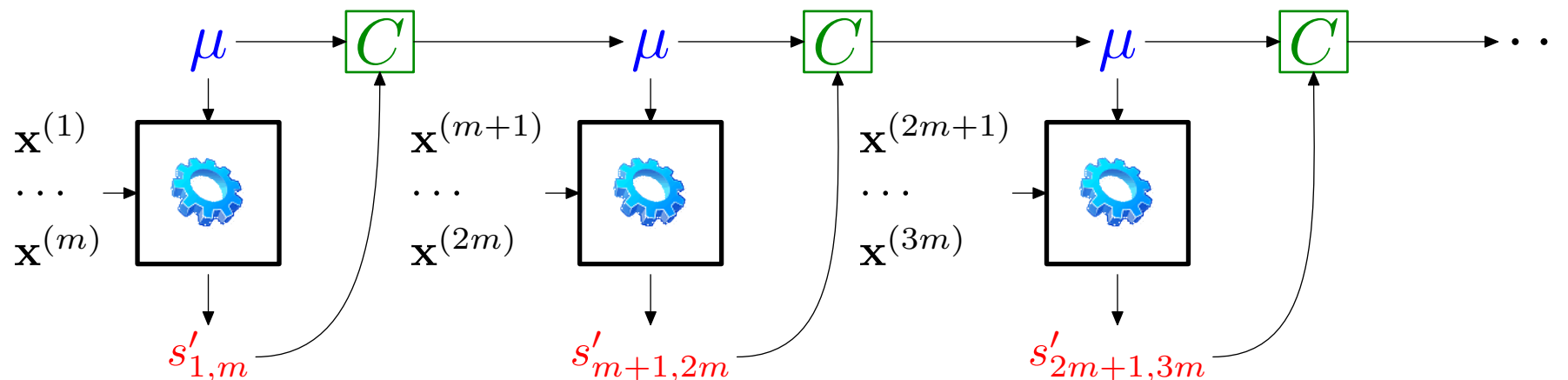


10 data points processed



4 data points processed

# Optimization parameter 2 of 2: minibatch size



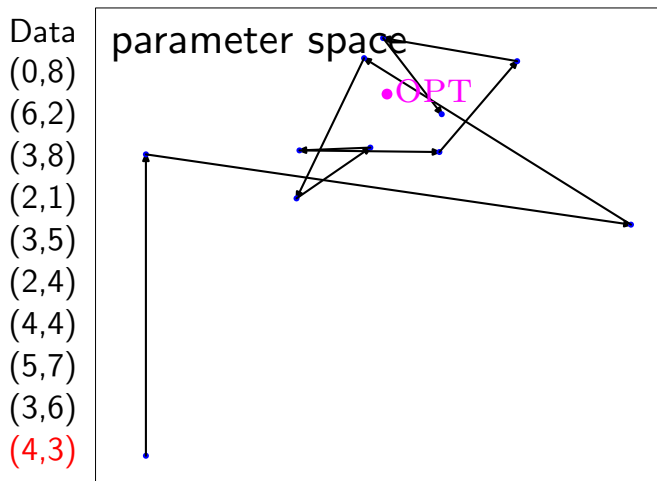
$m = \text{size of a mini-batch}$

$m = 1$

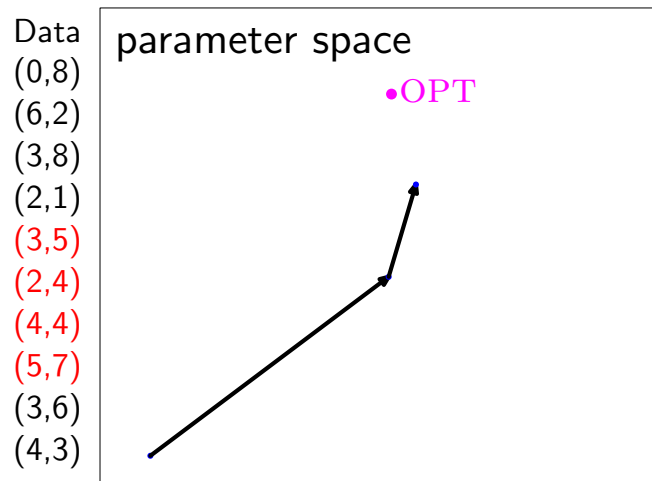
$m = n$

frequent updates, unstable

infrequent updates, stable

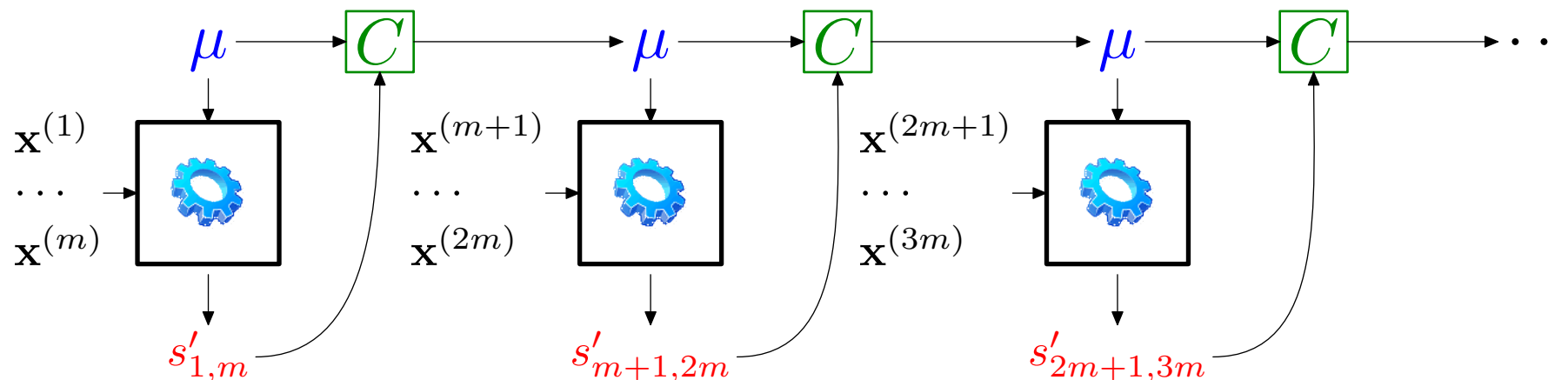


10 data points processed



8 data points processed

# Optimization parameter 2 of 2: minibatch size



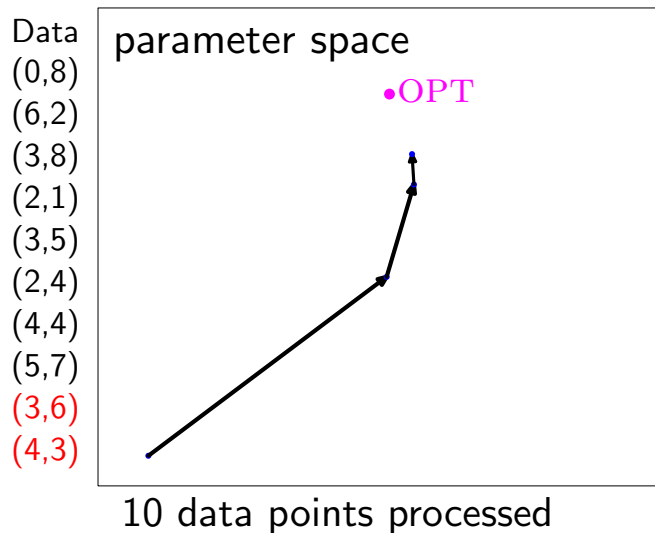
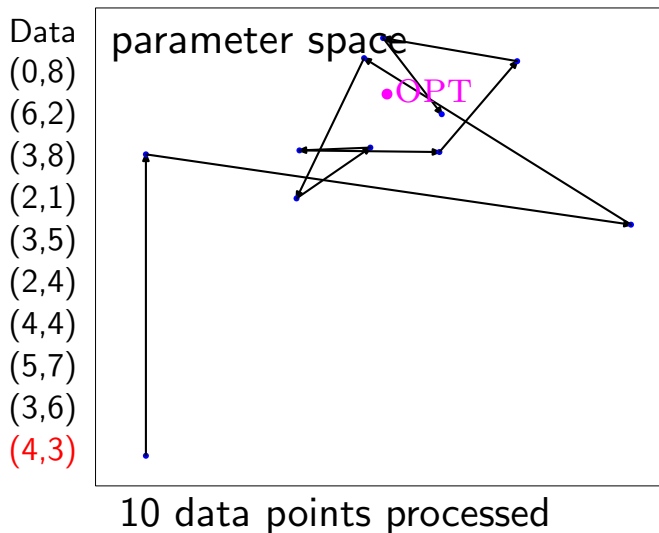
$m = \text{size of a mini-batch}$

$m = 1$

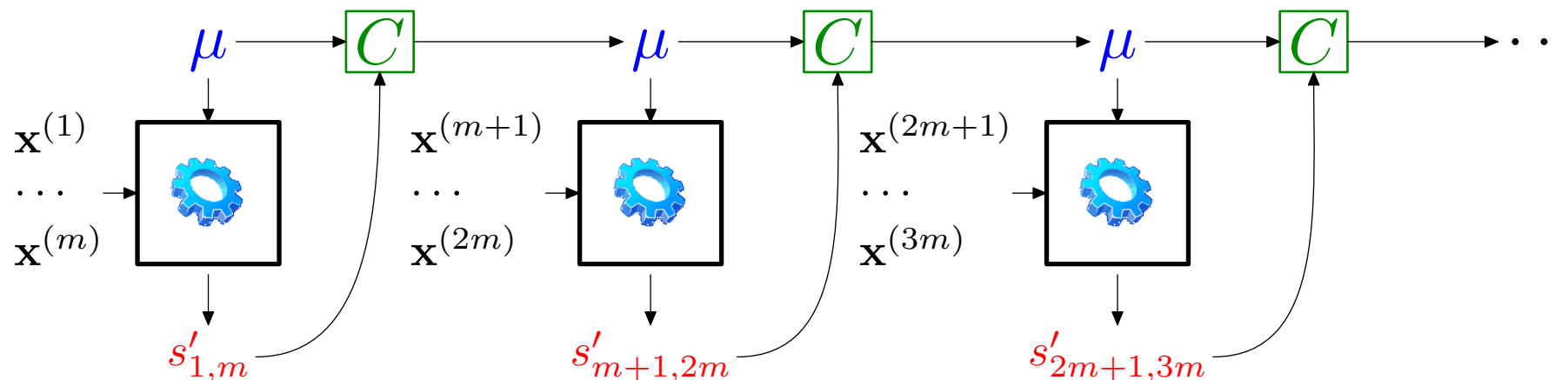
$m = n$

frequent updates, unstable

infrequent updates, stable



# Optimization parameter 2 of 2: minibatch size



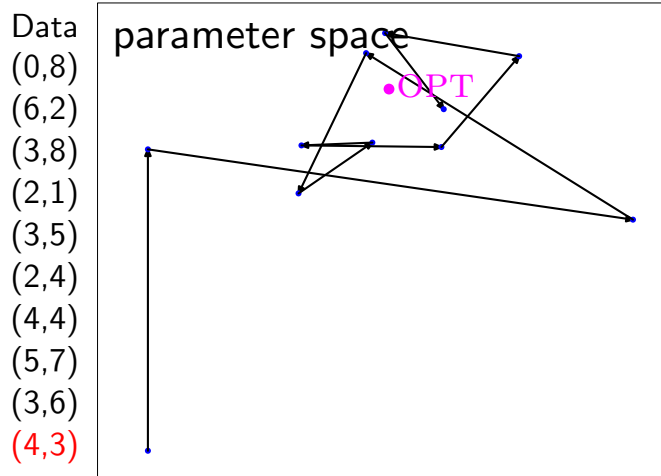
$m = \text{size of a mini-batch}$

$m = 1$

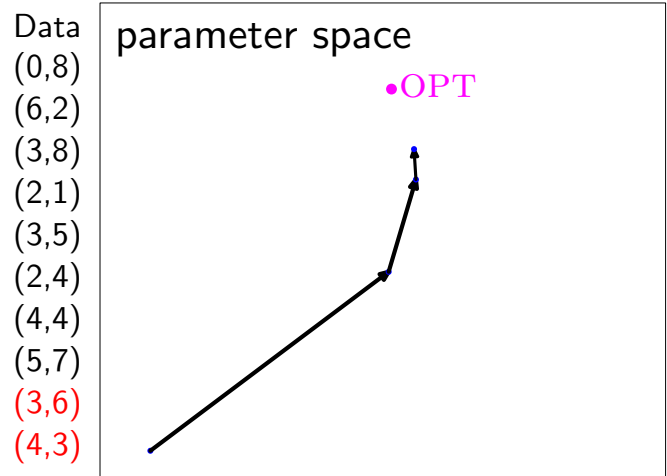
$m = n$

frequent updates, unstable

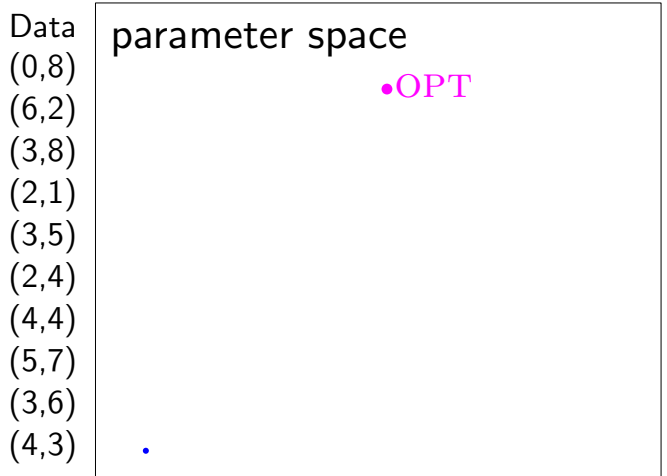
infrequent updates, stable



10 data points processed

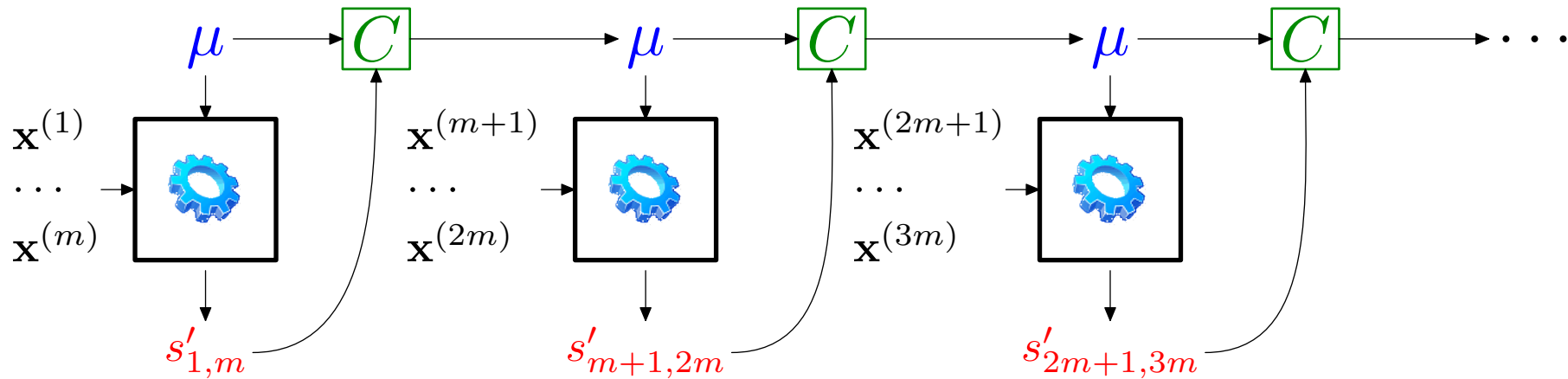


10 data points processed



0 data points processed

# Optimization parameter 2 of 2: minibatch size



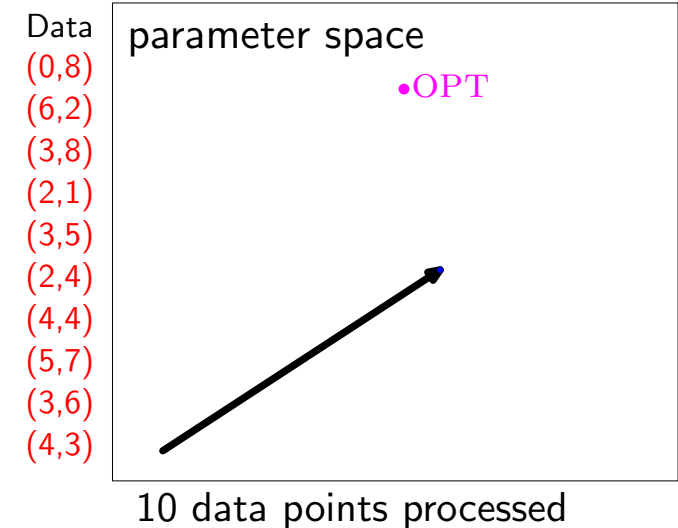
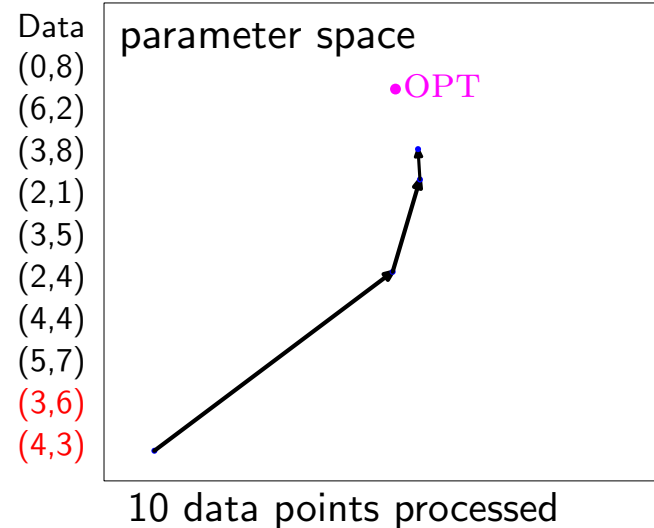
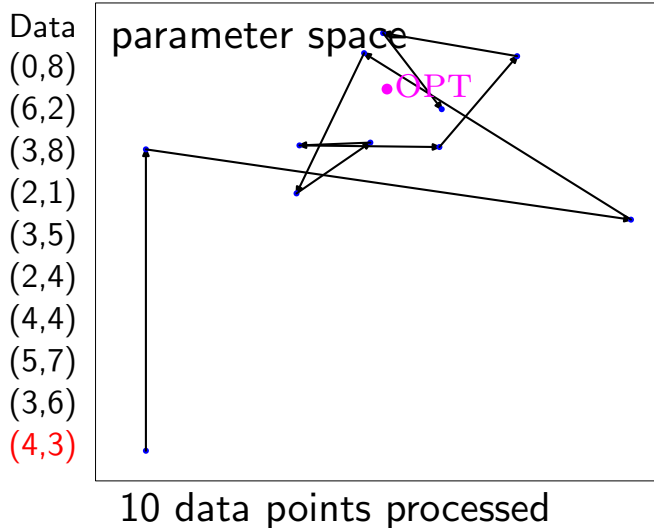
$m = \text{size of a mini-batch}$

$m = 1$

$m = n$

frequent updates, unstable

infrequent updates, stable



# Setting optimization parameters

stepsize reduction power  $\alpha$

mini-batch size  $m$

# Setting optimization parameters

stepsize reduction power  $\alpha$

mini-batch size  $m$

Document classification:

[Likelihood]

$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
0.5	-8.875	-8.710	-8.610	-8.555	-8.505	-8.172	-7.920	-7.906	-7.916
0.6	-8.604	-8.575	-8.540	-8.524	-8.235	-8.041	-7.898	-7.901	-7.916
0.7	-8.541	-8.533	-8.531	-8.354	-8.023	-7.943	-7.886	-7.896	-7.918
0.8	-8.519	-8.506	-8.493	-8.228	-7.933	-7.896	-7.883	-7.890	-7.922
0.9	-8.505	-8.486	-8.283	-8.106	-7.910	-7.889	-7.889	-7.891	-7.927
1.0	-8.471	-8.319	-8.204	-8.052	-7.919	-7.889	-7.892	-7.896	-7.937



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[Likelihood]

$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
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0.6	-8.604	-8.575	-8.540	-8.524	-8.235	-8.041	-7.898	-7.901	-7.916
0.7	-8.541	-8.533	-8.531	-8.354	-8.023	-7.943	-7.886	-7.896	-7.918
0.8	-8.519	-8.506	-8.493	-8.228	-7.933	-7.896	<b>-7.883</b>	-7.890	-7.922
0.9	-8.505	-8.486	-8.283	-8.106	-7.910	-7.889	-7.889	-7.891	-7.927
1.0	-8.471	-8.319	-8.204	-8.052	-7.919	-7.889	-7.892	-7.896	-7.937

# Setting optimization parameters

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mini-batch size  $m$

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[Likelihood]

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0.5	-8.875	-8.710	-8.610	-8.555	-8.505	-8.172	-7.920	-7.906	-7.916
0.6	-8.604	-8.575	-8.540	-8.524	-8.235	-8.041	-7.898	-7.901	-7.916
0.7	-8.541	-8.533	-8.531	-8.354	-8.023	-7.943	-7.886	-7.896	-7.918
0.8	-8.519	-8.506	-8.493	-8.228	-7.933	-7.896	<b>-7.883</b>	-7.890	-7.922
0.9	-8.505	-8.486	-8.283	-8.106	-7.910	-7.889	-7.889	-7.891	-7.927
1.0	-8.471	-8.319	-8.204	-8.052	-7.919	-7.889	-7.892	-7.896	-7.937

[Accuracy]

$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
0.5	5.4	5.4	5.5	5.6	6.0	25.7	48.8	49.9	44.6
0.6	5.4	5.4	5.6	5.6	22.3	36.1	48.7	49.3	44.2
0.7	5.5	5.5	5.6	11.1	39.9	43.3	48.1	49.0	43.5
0.8	5.6	5.6	6.0	21.7	47.3	45.0	<b>47.8</b>	49.5	42.8
0.9	5.8	6.0	13.4	32.4	48.7	48.4	46.4	49.4	42.4
1.0	6.2	11.8	19.6	35.2	47.6	49.5	47.5	49.3	41.7

# Setting optimization parameters

stepsize reduction power  $\alpha$

mini-batch size  $m$

## Document classification:

[Likelihood]

$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
0.5	-8.875	-8.710	-8.610	-8.555	-8.505	-8.172	-7.920	-7.906	-7.916
0.6	-8.604	-8.575	-8.540	-8.524	-8.235	-8.041	-7.898	-7.901	-7.916
0.7	-8.541	-8.533	-8.531	-8.354	-8.023	-7.943	-7.886	-7.896	-7.918
0.8	-8.519	-8.506	-8.493	-8.228	-7.933	-7.896	<b>-7.883</b>	-7.890	-7.922
0.9	-8.505	-8.486	-8.283	-8.106	-7.910	-7.889	-7.889	-7.891	-7.927
1.0	-8.471	-8.319	-8.204	-8.052	-7.919	-7.889	-7.892	-7.896	-7.937

[Accuracy]

$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
0.5	5.4	5.4	5.5	5.6	6.0	25.7	48.8	<b>49.9</b>	44.6
0.6	5.4	5.4	5.6	5.6	22.3	36.1	48.7	49.3	44.2
0.7	5.5	5.5	5.6	11.1	39.9	43.3	48.1	49.0	43.5
0.8	5.6	5.6	6.0	21.7	47.3	45.0	<b>47.8</b>	49.5	42.8
0.9	5.8	6.0	13.4	32.4	48.7	48.4	46.4	49.4	42.4
1.0	6.2	11.8	19.6	35.2	47.6	49.5	47.5	49.3	41.7

# Setting optimization parameters

stepsize reduction power  $\alpha$

mini-batch size  $m$

## Document classification:

[Likelihood]

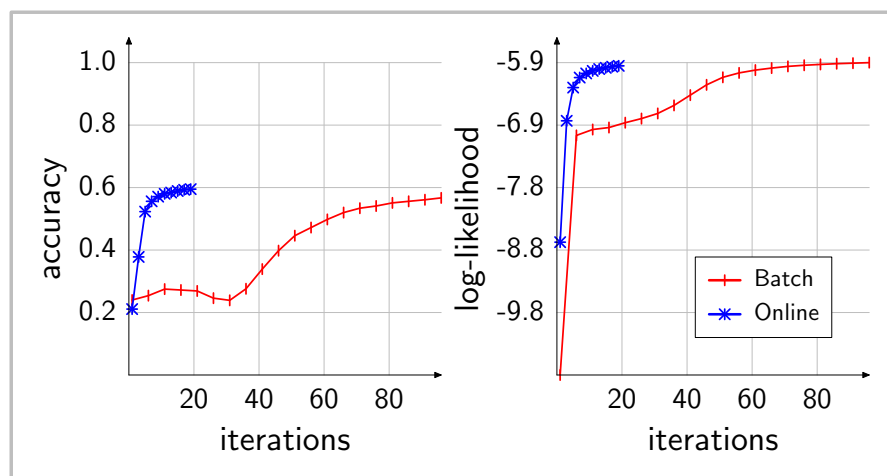
$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
0.5	-8.875	-8.710	-8.610	-8.555	-8.505	-8.172	-7.920	-7.906	-7.916
0.6	-8.604	-8.575	-8.540	-8.524	-8.235	-8.041	-7.898	-7.901	-7.916
0.7	-8.541	-8.533	-8.531	-8.354	-8.023	-7.943	-7.886	-7.896	-7.918
0.8	-8.519	-8.506	-8.493	-8.228	-7.933	-7.896	<b>-7.883</b>	-7.890	-7.922
0.9	-8.505	-8.486	-8.283	-8.106	-7.910	-7.889	-7.889	-7.891	-7.927
1.0	-8.471	-8.319	-8.204	-8.052	-7.919	-7.889	-7.892	-7.896	-7.937

[Accuracy]

$\alpha \backslash m$	1	3	10	30	100	300	1K	3K	10K
0.5	5.4	5.4	5.5	5.6	6.0	25.7	48.8	<b>49.9</b>	44.6
0.6	5.4	5.4	5.6	5.6	22.3	36.1	48.7	49.3	44.2
0.7	5.5	5.5	5.6	11.1	39.9	43.3	48.1	49.0	43.5
0.8	5.6	5.6	6.0	21.7	47.3	45.0	<b>47.8</b>	49.5	42.8
0.9	5.8	6.0	13.4	32.4	48.7	48.4	46.4	49.4	42.4
1.0	6.2	11.8	19.6	35.2	47.6	49.5	47.5	49.3	41.7

$(\alpha, m)$  important, but can set using likelihood (unsupervised)

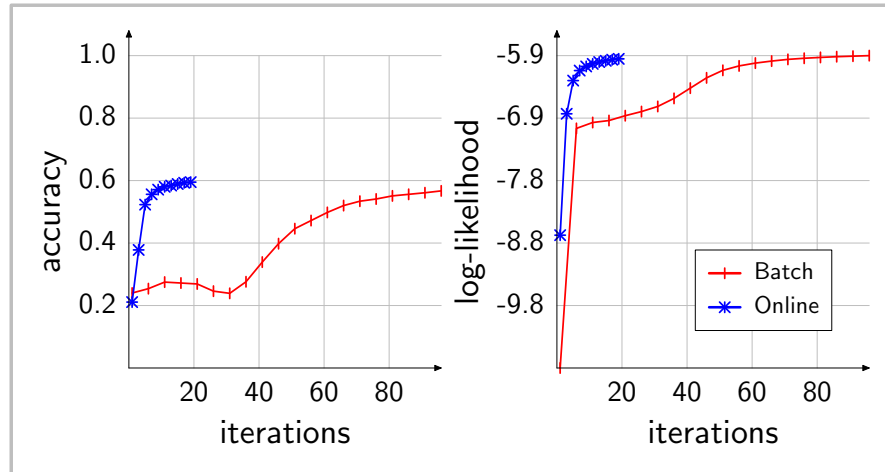
# Results: speed



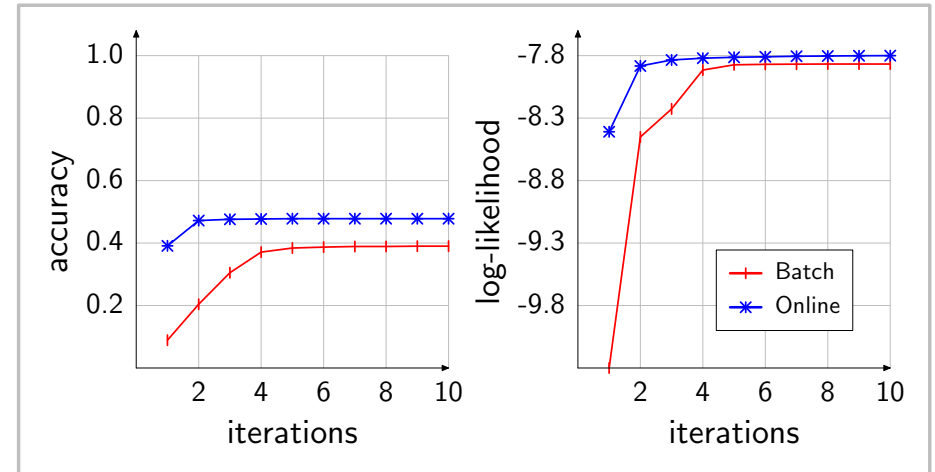
(a) POS tagging

Online converges faster than Batch

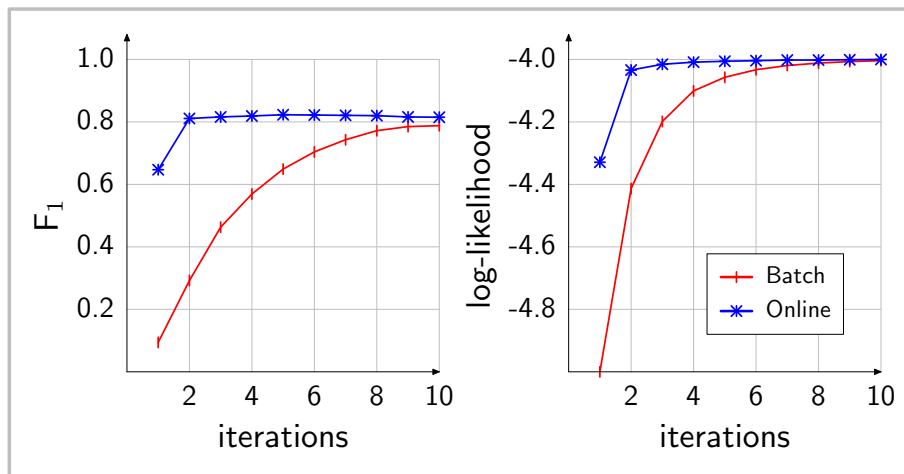
# Results: speed



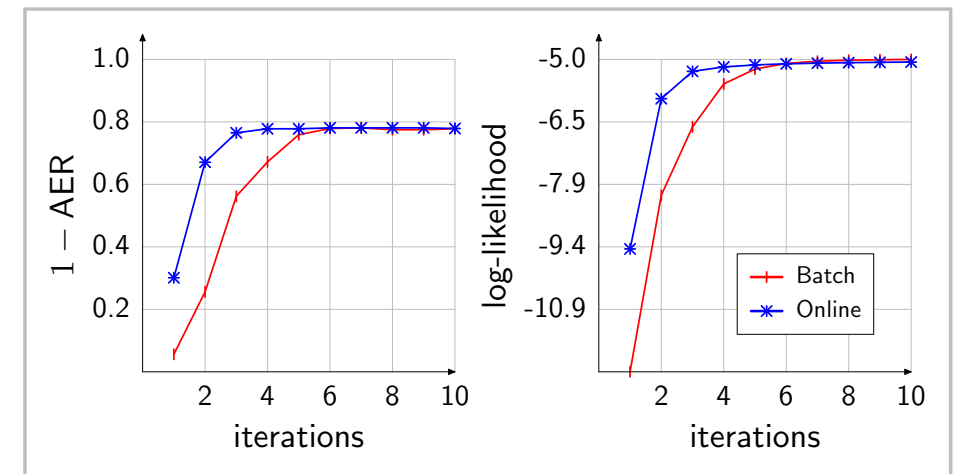
(a) POS tagging



(b) Document classification



(c) Word segmentation (English)



(d) Word alignment

Online converges faster than Batch

## Results: final accuracy

	POS	DOC	SEG	ALIGN
Batch	57.3	39.1	80.5	78.8
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Online: choose  $(\alpha, m)$  with highest likelihood

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### Mystery:

- Online EM obtains higher accuracy
- Batch EM and online EM optimize same objective function

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Two parts of optimizing non-convex objectives:

- (1) Find a good peak
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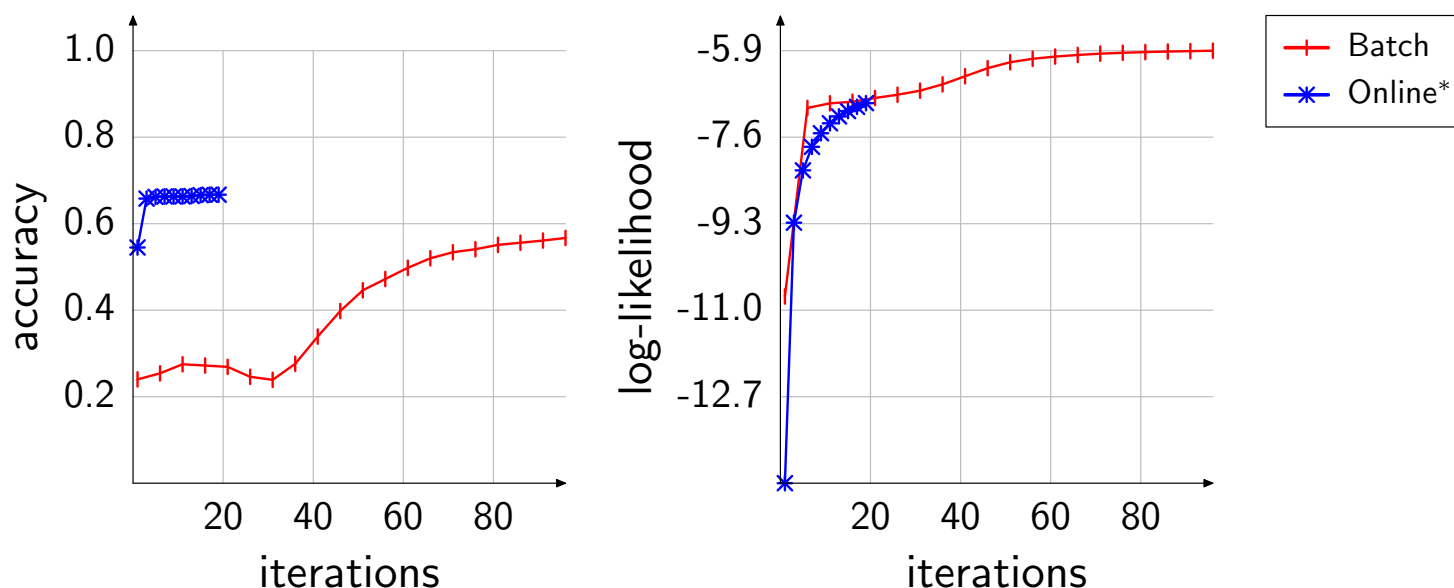
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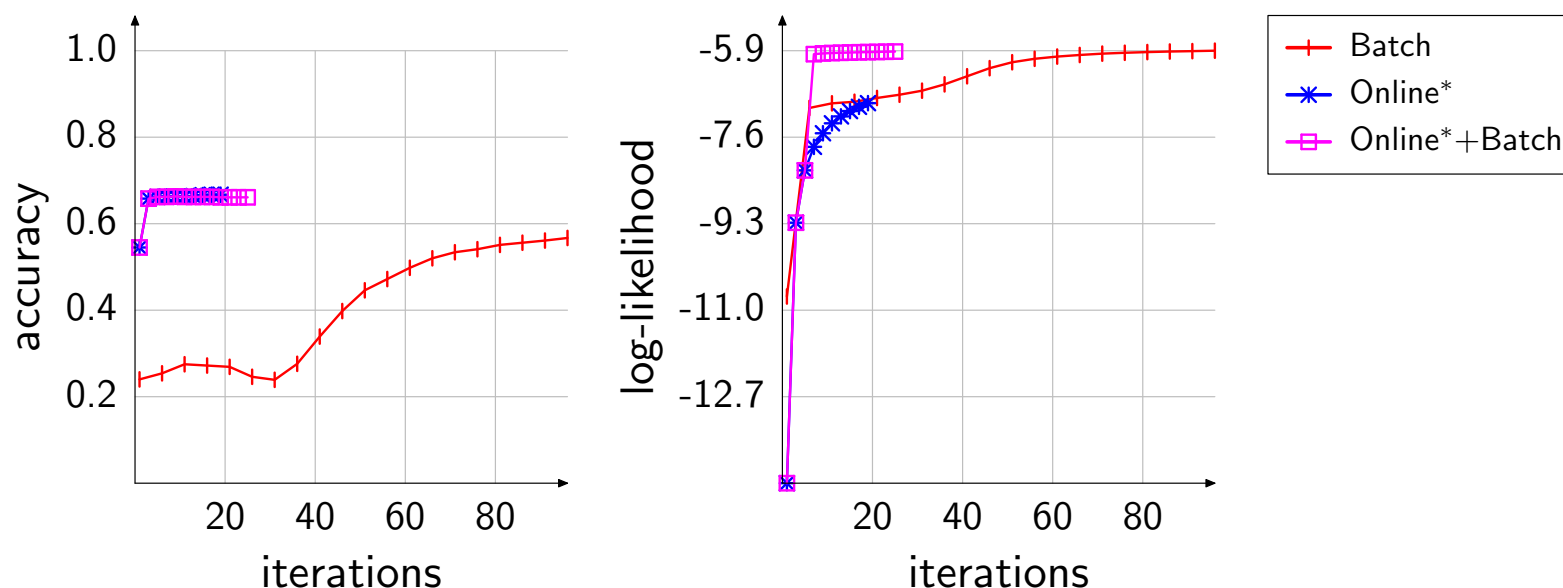
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Online\*+Batch: 5 iterations of Online\* then Batch

# Summary

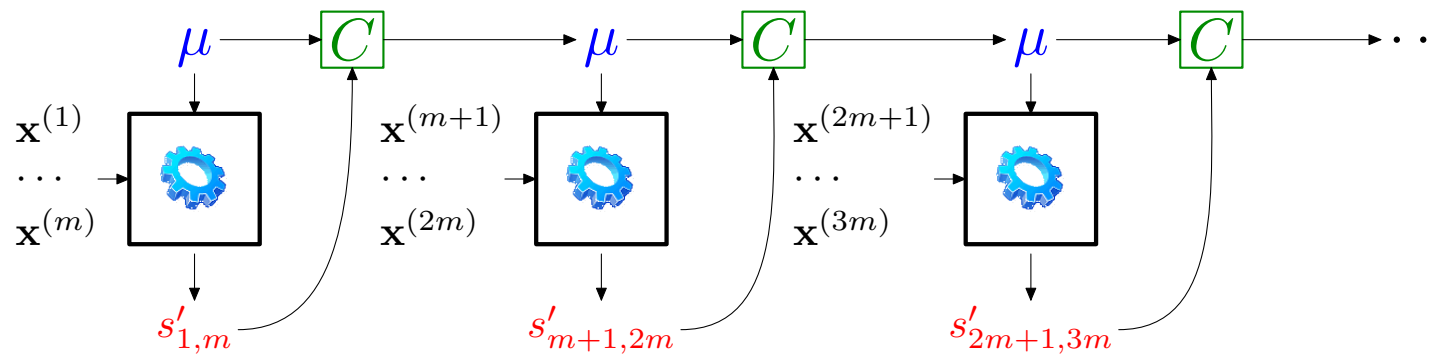
Goal: fast unsupervised learning



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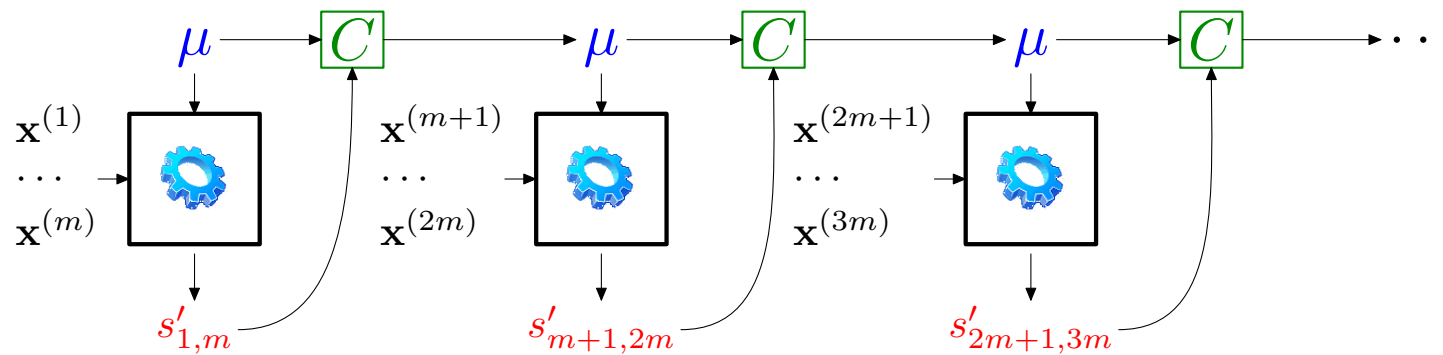
Online EM: update parameters more often



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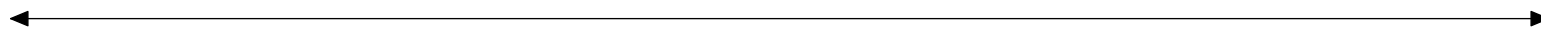
Goal: fast unsupervised learning

Online EM: update parameters more often



fast, unstable

slow, stable

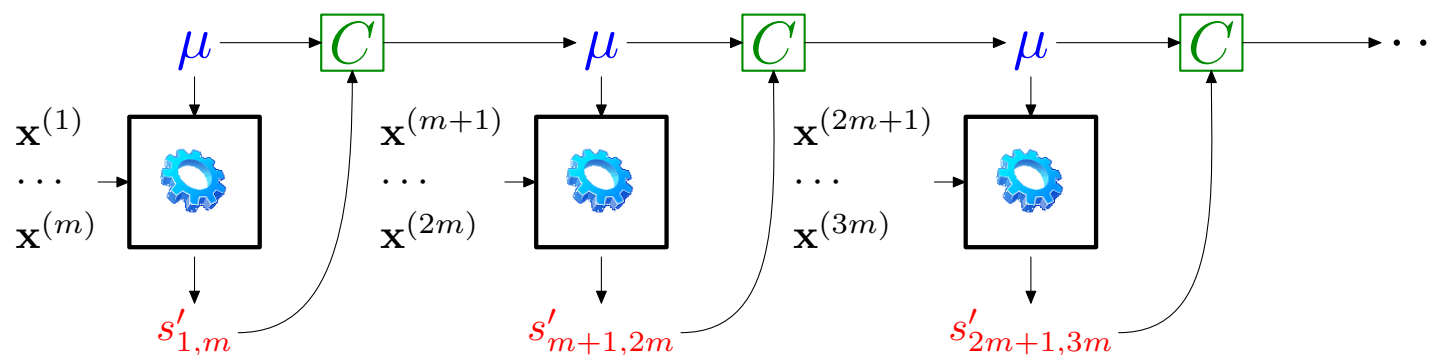


Stepsize and minibatches balance this tradeoff

# Summary

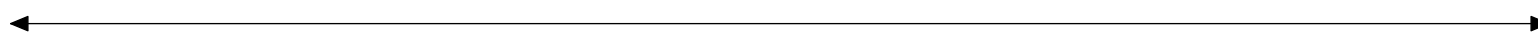
Goal: fast unsupervised learning

Online EM: update parameters more often



fast, unstable

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Stepsize and minibatches balance this tradeoff

Result: online EM is faster,  
and sometimes more accurate than batch EM