

minute summary

Problem: learning complex hidden-variable models Traditional solution: approximate EM \Rightarrow |M| one intractable model Our solution: product EM (train submodels to agree) two tractable submodels Applications: unsupervised NLP, phylogenetic HMMs

Motivating applications

Phylogenetic HMMs

Goal: model both nucleotide mutations across species and dependencies between adjacent sites



Computational challenge: doing inference in a loopy graph Agreement-based solution:

Break up model into the red part and the green part

Unsupervised word alignment

Goal: learn to output a matching between two sequences by modeling the translation process of words between a pair of sentences

Computational challenge: enumerating all matchings Agreement-based solution:



the railroad term is " demand loading le terme ferroviaire st '' chargement sur demande ferroviaire est chargement the railroad term is if "I demand loading

Agreement-Based Learning

Percy Liang, Dan Klein, Michael I. Jordan UC Berkeley
• Computer Science Division

Product EM

Setup:

M submodels $\{p_m(\mathbf{x}, \mathbf{z}; \theta_m) : m = 1, \dots, M\}$

Objective function:

 $\mathcal{O}_{\mathsf{agree}}(\boldsymbol{\theta}) \stackrel{\mathsf{def}}{=} \log \sum$

Interpretation:

Each submodel m independently generates $(\mathbf{x}_m, \mathbf{z}_m)$

$$\mathcal{O}_{\mathsf{agree}}(oldsymbol{ heta}) = p(\mathbf{x}_1 = \cdots = \mathbf{x}_1)$$

Algorithm:

Introduce auxiliary q, use Jensen's inequality:

 $\mathcal{O}_{\mathsf{agree}} \geq \mathcal{L}(\boldsymbol{\theta}, q) \stackrel{\mathsf{def}}{=} \sum \mathbb{E}_q \log p_m(\mathbf{x}, \mathbf{z}; \theta_m) + H(q)$

E-step: $q(\mathbf{z}) \propto \prod_m p$ M-step: $\theta_m = \operatorname{argmax}$



Properties:

- E-step couples submodels: could be intractable
- M-step decomposes into M tractable steps

Exponential family formulation

Assume submodels are in exponential family: $p_m(\mathbf{x}, \mathbf{z}; \theta_m) = \exp\left\{\theta_m^T\left(\phi_m^{\mathcal{X}}(\mathbf{x})\phi^{\mathcal{Z}}(\mathbf{z})\right) - A_m(\theta_m)\right\}$ for $\mathbf{x} \in \mathcal{X}, \mathbf{z} \in \mathcal{Z}_m$ and 0 otherwise Reformulation of Product EM:

 $\mu = E(b, \cap_m \mathcal{Z}_m) \stackrel{\text{def}}{=} \mathbb{E}_{q(\mathbf{z};b)} \phi^{\mathcal{Z}}(\mathbf{z}) \text{ with support } \cap_m \mathcal{Z}_m$

Aggregate parameters: $\mathbf{b} = \sum_{m} b_{m}, b_{m} = \phi_{m}^{\mathcal{X}}(\mathbf{x})^{T} \theta_{m}$ E-step: compute expected sufficient statistics M-step: set θ_m to match moments $\phi_m^{\mathcal{X}}(\mathbf{x})\mu$



$$\sum_{m} p_m(\mathbf{x}, \mathbf{z}; \theta_m)$$

- • \mathbf{z}_{M} (\mathbf{x}_M) $\mathbf{x}_M = \mathbf{x}, \mathbf{z}_1 = \cdots = \mathbf{z}_M; oldsymbol{ heta})$

$$p_m(\mathbf{x}, \mathbf{z}; \theta_m)$$

 $\mathbf{x}_{\theta'_m} \mathbb{E}_q \log p(\mathbf{x}, \mathbf{z}; \theta'_m)$



